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Carl Friedrich Gauss

THE MATHEMATICS TEACHER

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# Mathematical Training for Economic Thinking and Social Mindedness*

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WE OF TODAY ARE LIVING in a time of rapid technological advancement and of equally rapid social and economic change. In a democracy such as ours, desirable social reconstruction can come about only if people in general achieve an intelligent comprehension of the complex activities in which perforce they play a part. Hence the training of youth for effective economic thinking becomes an imperative obligation of education. The social-economic processes undergoing evolutionary changes before our very eyes may be regarded from two significant standpoints—their humanistic bearings, and their technical aspects. The technicalities of economic activities and social institutions are both qualitative and quantitative; the qualitative features are largely related to modern physical science, while the quantitative aspects involve statistical and mathematical methods.

It is here proposed to discuss some potentialities of appropriate mathematical instruction as yet quite unrealized in our American high schools. Conventional algebra and geometry, having but little

* Adapted from a radio address delivered over WNYC, February 16, 1934.

bearing upon worldly matters, are rather inadequate for an intelligent understanding of the vital problems that beset us today, and for that matter, the struggle for adjustment which must be faced by the rising generation. This writer believes that it is not only possible, but highly desirable, to *arrange mathematical instruction for high school boys and girls so that as adults they will be able and willing to think about social-economic problems in quantitative terms.* It is his firm conviction that there is a crying need for substantial training in what might appropriately be called "quantitative thinking." The average intelligent man or woman does not ordinarily possess any great degree of sensitivity to the *quantitative* aspects of relationships; it is their *qualitative* side that makes by far the greater and more natural appeal. Sensitivity and discrimination with regard to numerical and quantitative matters, not quite so obvious, needs to be cultivated, and can be best secured through purposeful training and education. We shall illustrate what is meant by "sensitivity to quantitative thinking" by pointing out several specific concepts which are essential to clear thinking about social and economic matters.

Consider, for example, the delicate social problem which at this very moment is engaging the attention of certain peoples, the problem of sterilizing so-called unfit human beings. This involves, among other things, defining exactly what is meant by unfit, which in turn depends upon the precise determination of who is a "moron," an "imbecile," an "idiot," and so on. Now this can be done on the basis of a frequency distribution. That is, loose thinking and emotionalism can be avoided by grasping the meaning and significance of a statistical distribution of the frequency with which certain items appear in a group. We should really like to know, out of one thousand individuals taken at random, how many are morons? How many are imbeciles? Vague qualitative thinking frequently leads to inconsistent or absurd conclusions. Thus where the cautious person says *sometimes*, the dogmatic person will say *always*, the optimist will say *often*, and the skeptic, *never*. Please note that although these terms vaguely suggest quantitative distinctions, they are nevertheless substantially qualitative and subjective. Rigorous, intelligent and constructive thinking results only when mathematical and statistical methods are used. How much more effective to say that of one thousand individuals taken at random we would find 130 dull persons, 60 border-line cases, and

only 10 feeble-minded; and furthermore, that of these 10 feeble-minded, 75% were morons, 20% were imbeciles, and only 5% were hopeless idiots. How different the picture when it is realized that by and large only *one* person out of *ten thousand* will be found to be an idiot! Subjective animosities and emotional prejudices tend to disappear as greater use is made of quantitative methods.

Let us consider another quantitative concept involved in many social and economic problems,—the concept of probability. What vagueness of thought is usually associated with the element of chance! Thus the degree of certainty of the occurrence of a given event, or series of events, is often inadequately stated only qualitatively, as in the expressions: "It may rain," "it is very likely to rain," "perhaps it will rain," "no doubt it will rain," or "probable showers tomorrow." More precisely, just *how* probable? Exactly how likely will a man of forty live to become forty-one years old? What is the exact chance that if I cross the street in the middle of the block, I will be hit by an automobile? What is the precise numercial measure of the probability of the occurrence of a fire, an accident, a tornado, a sickness, or other contingency? Questions such as these can be answered intelligently and effectively, but only on a strict, mathematical basis. That great bulwark of modern society, life assurrance, rests not on qualitative assumptions, but on the solid foundation of what are technically known as actuarial methods, special departments of mathematics and statistics based upon the concepts of probability and life contingencies. Many other vital economic and social institutions are related to the concept of probability, including fire insurance, all forms of casualty and automobile insurance, annuities, pensions, retirement incomes, health and sickness insurance, workmen's compensation, unemployment insurance, and so on. Technical knowledge of probability and statistics is also of great value where prediction or forecasting of events is desirable, as in the incidence and control of diseases, safety campaigns, accident prevention, trends in birth and death rates, prevalence and reduction of crime and racketeering, the study of suicides and homicides, morbidity and disability rates, periodic health examinations, growth of population, and trends in the shift from rural to urban centers. This very incomplete list will serve to suggest the wide range of possible usefulness.

Consider a third important quantitative concept, the significant notion of a rate of change. The mere qualitative fact that a quantity

*does* change is obvious and superficial; what matters a great deal more, however, is: just *how* is it changing? Many an issue becomes sadly obfuscated through failure to distinguish between, let us say, a *high* price and a *changing* price; or between a *low* birth rate and a *falling* birth rate; or between a *huge* public debt and a *rapidly increasing* public debt. Oftentimes it is far more essential to know how rapidly a quantity is increasing or decreasing at a given time or during a given interval than it is to know how large or how small it may be. Such knowledge can be gained by an appreciation of the methods of the calculus, which, despite their formidable and fearsome name, can, nevertheless (in elementary fashion, to be sure) be made quite intelligible to the average high school youngster. It would almost seem superfluous to suggest the many social and economic phenomena to which this rate-of-change concept can profitably be applied. To mention just a few, there are the changes in the price of commodities; the fluctuation of various index numbers; the trend of the cost of living, of marriage and divorce rates, of infant mortality: variations in real estate and land values, in tax rates, and in public incomes and expenditures; the progress of manufacturing and industrial enterprises, the flow of imports and exports, the volume of business activities; changes in salaries, wages, and incomes; the rise and fall of interest rates and security prices; and such barometers as the number of business failures, the amount of brokers' loans, weekly freight car loadings, monthly electric power consumption, or the volume of department store sales. Indeed, it is no exaggeration to say that the bulk of economic, industrial and social activities of a complex, civilized society can be properly understood only in terms of the rate of change concept.

A fourth concept of great significance, and one which bears directly on the rate-of-change idea, is the notion of functionality—that is, the dependence of one quantity upon another, or the nature of the relationship between two or more quantities so connected that a change in the magnitude of one of them causes some sort of a change in the value of the others. This interdependence of mutually related variable quantities is commonplace. Thus the price of a commodity depends jointly upon the nature of the supply and demand curves for that commodity; the license fee I pay for my automobile depends in a definite quantitative way upon the gross weight of the car; the taxes I pay on my house depend upon

the assessed valuation of the property as well as upon the current tax rate; or the amount of the premium on my life insurance depends upon the face value of the policy, the age at which it was issued, and the mathematical nature of the benefit. Such variations between related quantities are of several different kinds or types. The simplest, of course, is direct variation, where as one variable increases or decreases, the other also increases or decreases, respectively, in direct proportion; thus if the interest on $100 is $5, then on $200 it is $10, on $300 it is $15, etc. Another common type of variation is the inverse type, where as one quantity increases, the other decreases in some definite manner, as, e.g., when the pressure on a gas is increased, its volume is decreased, and conversely. Other types of variation are not quite so simple. Thus the speed of a ship and its resistance, or the amount of fuel consumed by a locomotive at various speeds, are more complicated relations following some form of power law. Other interesting variations are those periodic functions or cyclic changes which reveal the effects of seasonal influences, such as the variation of electric power consumption, or the incidence of suicides. Still other functions exhibit logarithmic or exponential variation, where the rate at which a quantity changes at any given instant itself depends upon the particular magnitude of the quantity at that instant. Familiar illustrations are the growth of population, or the accumulation of money at compound interest. Finally, many functional relationships follow more complicated laws of variation which it is difficult, though not impossible, to express mathematically, as e.g., the fluctuations of blood pressure with age, or the mortality rate of human beings.

We come finally to a group of highly important ideas which have heretofore been largely neglected in educational programs, although without quantitative treatment it is well nigh impossible to appreciate their full significance. I am thinking of the concept of compound interest. Nowhere in their educational experience do young people find the opportunity to become familiar with the fundamental distinctions between money and capital, between debt and equity, or between present value and future value; or the difference between paying a debt in a lump sum and extinguishing the debt by amortization. Neither do they become oriented in the notion of the capitalized value of an expenditure, or in the nature of the depreciation of an asset. All of this must be considered a distinct

loss. How essential it is for the well-informed man or woman of today to be conversant with such matters. Many of these ideas strike right at the heart of questions that are being debated from day to day, and which touch the daily lives and affairs of all of us.

Let us examine one or two of them for a moment. Consider how much larger the significance of an expenditure looms up when it is realized that the real cost of many articles is not simply their purchase price, but in reality the capitalized value of the sum paid, which means the amount to which that sum would accumulate if invested instead. At 5% compound interest any sum of money doubles itself in about 15 years. Or, looking at it another way, the capitalized value of a $100 purchase, at 5%, is $2,000. This concept of capitalized value is closely related to the notion of the depreciation of an asset. As time goes on, the value of an article becomes less than its original purchase price, whether this is due to obsolescence, wearing out, or whatever reason. But all assets do not depreciate at the same rate. Some, like an automobile, depreciate very rapidly in the early period of their life, others depreciate more or less uniformly, and still others, like a vacuum cleaner, depreciate relatively slowly in the beginning and much faster toward the end. These are important differences, and are only adequately treated by appropriate mathematical methods. Again, take the concepts of amortization and annuities; whether we are considering insurance premiums, or installment purchasing, or Home Owners' Loan bonds, the principle of amortization is involved. And behind these ideas lies that most fundamental principle of all, the law of compound interest growth.

The few concepts alluded to above are only the broad underlying principles involved in discriminating, rigorous, quantitative thinking. Many other related, though no less important concepts readily suggest themselves. Thus in a connection with frequency distributions, there are the matters of collecting data; sampling; graphic representation; measures of central tendency, such as mode, median, mean; the matter of spread, or dispersion; the use of weighted averages and index numbers; correlation; various types of distributions, including the normal binomial distribution curve, cumulative frequency curves, and the so-called Lorenz curve, which reveals with startling clarity and forcefulness inequitable distributions, whether of wealth or of incomes, or anything else. In

connection with probability, we have the associated ideas of the normal probability curve, the curve of error, the precision of measurements, as well as vital statistics, crude death rates and mortality tables. Along with the idea of rate of change concept, there is the converse problem, given the rate of change between two variables, to determine the functional relational between them; also, the contrast between an exponential law and a power law. With respect to the function concept, we should consider also the nature of growth curves and of ratio and difference charts; empirical functions; finite differences; curve-fitting; and the determination of the law of connection between variables. And finally, in conjunction with investment and finance, some familiarity with bond yield, bond tables, building loans, discounts, equation of value, and the nature of life insurance premiums and reserves is desirable.

It is sincerely believed that it is entirely possible for high school pupils to secure at least some understanding, even if only an initial orientation, in most, if not all of the concepts here mentioned; and if this contention is substantially true, then this writer, for one, cannot justify the current superficial tinkering with the external and inconsequential details of secondary mathematics curricula. In so doing we are losing a priceless opportunity to put mathematical instruction on an adequate basis where its popularity and value, instead of being open to question, would constantly be enhanced. It is not necessary that our young people be able to solve the problem of the industrial engineer who, in using the formula "sales÷production = 1," wondered what it meant when, during the period of depression, it approached the value $0 \div 0$. But I see no reason why, on the other hand, they should not be able to realize that a reduction in yield on an investment from 4% to 3% is equivalent to a reduction of 25% in income, or why the premium on an endowment policy is so much greater than on a similar whole life policy, or why the basic tax rate on property needs to be expressed as 0.0232233494 instead of simply 0.02.

Children who can achieve reasonable results in algebra, geometry, Latin or advanced biology can probably also profit by mathematical instruction when it has been appropriately organized to facilitate quantitative thinking. It is the writer's hope, in the near future, to be able to present in concrete form a proposed organization of materials of instruction to accomplish this purpose.

### Selected Readings


1. Bauer, G. N. *Mathematics Preparatory to Statistics and Finance.* Macmillan, 1930.
2. Evans, G. C. *Mathematical Introduction to Economics.* McGraw-Hill, 1930.
3. Fisher, Irving. *The Application of Mathematics to the Social Sciences.* National Council of Teachers of Mathematics, Sixth Yearbook, 1931, pp. 1–17.
4. Friend, M. R. *Earning and Spending the Family Income.* Appleton, 1930.
5. Hamley, H. R. *Relational and Functional Thinking in Mathematics.* National Council of Teachers of Mathematics, Ninth Yearbook, 1934.
6. Hart, Wm. L. *The Mathematics of Investment.* National Council of Teachers of Mathematics, Sixth Yearbook, 1931, pp. 61–85.
7. Lietzmann, W. *Funktion and Graphische Darstellung.* F. Hirt, Breslau, 1925.
8. Lietzmann, W. *The Function Concept and Graphical Methods in Statistics and Economics.* National Council of Teachers of Math., Seventh Yearbook, 1932, pp. 74–92.
9. Lovitt, W., and Holtzclaw, H. *Mathematics of Business.* Appleton, 1926.
10. Metzner, Karl. *Staatsbürgerkunde im mathematischnaturwissenschaftlichen Unterricht.* Quelle und Meyer, Leipzig, 1931.
11. Moir, Henry. *Life Assurance Primer.* Spectator Co., (3rd ed.), 1921.
12. *Population Trends in the United States.* Hoover Monograph, Recent Social Trends, McGraw-Hill, 1933.
13. *Recent Social Trends in the United States.* Report of the President's Research Committee on Social Trends. McGraw-Hill, 1933.
14. Schlauch, W. S. *Mathematics and Measuring of World Trends and Forces, A.D. 1932.* National Council of Teachers of Mathematics, Seventh Yearbook, 1932, pp. 119–136.
15. *Statistical Abstracts of the United States.* U. S. Dept. of Commerce. Gov't. Printing Office, Washington, D.C., 1931.
16. Weinberger, O. *Mathematische Volkswirtschaftlehre.* 1930.
17. Zühlke, Paul. *Politische Mathematik. Zentralinstitut für Erziehung und Unterricht.* Mittler, Berlin, 1923.

# Mathematics in the Integrated Curriculum*

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This paper very properly begins with a definition. Integration, as we all know, means to make into a whole, or to combine into a completely related unit. From the standpoint of a school program it means that the student not only has learned many things in school but he should also have acquired a unified idea of all the parts and of their relations to one another and to topics outside. It is an aim of education and a worthy one. Teachers everywhere hope that the individuals whom they teach will eventually be able to react to most life situations with intelligence and skill.

Three attempts to attain this objective are discussed. First, for many years children and adults were taught separate subjects with the hope and fond belief that when they finished they would be prepared to meet the problems of life. Second, of late years teachers have been attempting to use the natural correlations between fields of knowledge and parts of the same field in order to add interest and increase the likelihood of successful application when needed. They have not, however, withdrawn from the position which holds that the individual may best attain a workable, logically organized grasp of certain essential sections of fields of knowledge by a period of intensive study on the topics treated. Third, more recently a certain group of leaders in the American school system are proposing a curriculum made up of units of work which may continue as long as a year, and in which subject matter from any field may be used.

*Separate Subject Teaching.* The immediate interest of this paper is in the place of mathematics in these schemes. The first two are discussed rather briefly. Separate subject teaching is familiar to all of us and is all too common today. The pupils learn much about separate subjects but see little connection between them. As a result the pupils are in many cases unable to apply their knowledge

* Read on February 9, 1934, at a meeting of the Oklahoma Section of the Mathematical Association of America at Oklahoma City.

in particular situations. That is, their knowledge is not integrated. Mathematics as well as other subjects, is taught in what has been called "water-tight-compartments"; algebra is one subject, geometry is another, and so on. Sometimes pupils see connections between the parts of the field, but probably more often they do not. Integration, as defined above, is a rare thing rather than the rule.

*Correlated or General Courses.* The movement for correlated or unified courses in the junior high school, with which we are all more or less familiar, was initiated for the purpose of remedying these faults and others. Dr. Newlon of Columbia University says:

> The earliest experimentation (with unified courses) was with mathematics, but the first successful attempt at unification was in general science. It was felt that youth of the junior high school age should have a practical and interesting introduction to science. The most successful general science courses were built around scientific problems closely connected with the every day experience of children that involved not one science but several. Many conventional minded science teachers considered such a program utterly unthinkable, but the idea had a strong popular appeal and in twenty years has certainly demonstrated its validity. General science courses are common now in secondary schools throughout the entire country. About a decade ago this principle was introduced in the social sciences, particularly at the junior-high-school level. So called "unified" social science courses, organized around broad themes or problems of contemporary life according to various plans, gained widespread acceptance, and it would look as if history were about to repeat itself and the unified social science programs become as popular as the general science offerings. In recent years a more satisfactory integration of mathematics subjects has been effected at the junior-high-school level, and the general mathematics course is now accepted in all schools that make any pretense to being progressive. This tendency is now descernible in the offerings of a number of senior high schools.[1]

We have seen a similar development in college courses in recent years. The courses in general mathematics attempt so to fuse or correlate the subject matter that the student gets the real value as well as the relations of the various parts of the subject, but at the same time gets a rigorous, connected discourse.

The new psychology of teaching has placed emphasis "upon that education which will meet the needs of the people of today without reference to the subject matter which has been traditionally taught,"[2] but measurements of results of teaching general mathe-

[1] J. H. Newlon, "The Tendency toward Integration in the High School Curriculum." *Junior-Senior High School Clearing House.* VII: 397–398.

[2] Clarence McCormick, *The Teaching of General Mathematics in the Secondary Schools of the United States.* Teachers College Bureau of Publications, 1928. p. 1.

matics seem to indicate that pupils are learning as much or more mathematics than they did by the use of the older type of curriculum. College students who have had two or more years of general mathematics in high school seem to make slightly better grades in, and to have a better liking for, college mathematics.[3]

*The Use of Integrated Units of Work.* The third discussion deals with an attempt to attain the aim of integration of knowledge by using a radically different type of school organization and curriculum. This type of curriculum is generally made up of a unit of work, or a series of such units, for each school year. A discussion of a school system organized on this basis is included to clarify the idea. In the Lincoln Elementary School of Teachers College, Columbia University, at the beginning of each year each room chooses, with the aid of the teacher, some big topic about which practically the entire work of the year may be built. For example, one second grade chose the topic *Carrying the Mail.* Although the children were free to read or do anything in which they were interested, it was desirable that their activity be limited to the topic being studied. In this way, under the direction of a skillful teacher, the children learn many useful things. The great advantage is that the children are intensely interested in and see a reason for acquiring skills and knowledge. Other topics which have been chosen are *Adventuring With Toys, Indian Life and the Dutch Colonial Settlement,* and *Western Youth Meets Eastern Culture.* It is easy to see how these topics might be expanded to include much of the subject matter in the elementary school curriculum. The work done by the pupils and the teacher includes work from practically all subject matter fields and is all related to the particular topic chosen. The year's work as a whole is called a Unit of Work or an Integrated Unit.

Notice how radically it differs from the conventional procedure found in our own schools. The pupils do not have an arithmetic lesson, or a spelling lesson, or any of the others. If they need to or want to know where Madison, Wisconsin, is or what is produced in Idaho, this is learned, but no geography as geography is studied. The pupils read books about how mail is carried, and perhaps they study a geography to see the type of country some particular route traverses; they may make models of old mail coaches or of modern mail planes. They learn that regular mail routes were

[3] Clarence McCormick, Op. cit. pp. 51–52.

extended to California at the time it became a state. They also make written and oral reports on related topics. Many other things might be brought into the unit until it included more than a child will ordinarily learn in one year.

A school program made up of units of work of this sort is often called an integrated curriculum. The main theme or topic serves as the integrating factor; that is, all the child's experiences are related to this big idea, and the assumption is that since he has received his knowledge as a related whole in this way it will be more usable in other situations. This paper is concerned largely with the place of mathematics in this sort of curriculum, and suggests what will be necessary in order that the program may have the approval of the teachers of mathematics.

*The Extent of the Use of the Integrated Curriculum.* The same method, or organization, has been proposed for both the junior and senior high schools. It has not proceeded so far, however. In most cases in the secondary school an integrated unit of work uses subject matter from two or more fields while other subjects are taught as usual. For example, in the Lincoln School a unit on *Western Youth Meets Eastern Culture* used materials from the fields of social science, English, and art. Often work in physics and mathematics is taught together. Similar work is being done in some of the of the high school subjects; but, as yet, it has had little effect. Some discussion of the tendency in this field is included later. Although orientation or integrated courses have been organized in many colleges, no discussion of that phase of the subject is included here.

*Results of the use of the Integrated Curriculum.* An examination of the printed reports of such teaching seems to indicate that pupils learn more of some things and less of others than when taught in the ordinary way. Much more advance is made in reading than in other subjects; the advance in spelling and arithmetic, however, is usually less than is expected in an ordinary school. What the pupil learns in any particular subject has another disadvantage. It is learned piecemeal; the pupils may have a large number of facts but will probably have no logically organized knowledge of the topic. No data are available to show what pupils taught by this method in the junior and senior high schools have attained in particular subjects. It is too early to pass judgement on that phase of the program.

It should be pointed out that teachers and administrators who adhere to this method of teaching are more interested in other aims of education than in the mere learning of subject matter. In extreme cases it seems that they care nothing about what the pupil *knows* when he is through. All will admit, of course, that other aims, such as learning to think clearly, learning to consider all possible conditions before making a decision of any kind, learning to be tolerant of others, and the like are quite as important.

*Place of Subject Matter Content in the Integrated Curriculum.* The above discussion suggests a more careful analysis to see just what is done in regard to subject matter in the integrated unit. "An examination of a large number of courses of study, and also integrated units prepared independently, reveal that, while the authors may subscribe theoretically to the idea of using only the subject matter that interests children, a feeling of compulsion exists which forces them to make provisions for the attainment of certain skills and informations by the pupils. This idea may be due to their experience in teaching the older type curriculum, or it may be due to certain outside pressures.

"Many individual teachers have published accounts of their use of certain integrated units. A number of these teachers frankly admit that they guide the choices of activities of their pupils to fit the desires of the teachers, and they also admit that there are certain desirable skills and informations which the pupils should have. These skills and informations will not necessarily come from the activities of the particular class in pursuing the unit."[4]

*Disadvantages of the use of the Integrated Curriculum.* "The use of the integrated unit has certain dangers which should be pointed out. The results obtained by the use of this method depend, as in other methods of teaching, on the type of teacher using the method. First, an inexperienced teacher has no rich background in teaching and in the subject matter to be taught, and probably he will not consider the subject matter valuable and neglect it more than the older teacher. This is a problem of teacher training; the young prospective teacher must know the content well because, even though he is not to attempt to include it in the curriculum, he must be able to select. In the other type of teaching it is selected

[4] J. H. Zant, *The Teaching Plan for the Unit of Work in Junior High School Mathematics*. Guthrie, Oklahoma, Co-operative Publishing Company, 1934. pp. 12–13.

and organized by the textbook writer. Second, the teacher who has had little preparation, whether he is experienced or inexperienced, will have neither the pedagogical basis for the selection of subject matter, skills, and the like, nor a wide knowledge of the materials of the subject from which the selection is to be made. Third, certain experienced teachers will be without any adequate preparation in some of the subjects. This is more especially true in the junior and senior high school grades. In these grades the teachers have taught or prepared to teach certain subjects, like English or history. If they are required to develop and teach an integrated unit, some subjects will be neglected while others will be overstressed. Centering the entire curriculum around a social science theme is likely to neglect some of the necessary skills, informations, and appreciations in mathematics or some other field, merely because the teacher does not know enough about these subjects to see all the significant relationships. This is probably the most serious fault in this method of teaching. Fourth, advisers and directors of curriculum revision programs may have little knowledge of or sympathy with the aims of the separate subjects."[5]

*Proposals in regard to the use of the Integrated Unit in Secondary Schools.* As stated above, this method of teaching has been little used in the high school, but leaders in this field are predicting its rapid expansion. Dr. Newlon has said recently, ". . . a new interest in this approach to curriculum construction is clearly discernible in the high school."[6] Definite proposals in regard to the use of the integrated curriculum in the secondary school have been made. Three of these are discussed as typical. The first is that made by Dr. Will French, Superintendent of Schools, Tulsa, Oklahoma.

He suggests that "much of our present advanced practice is but a superficial attack on the whole problem of revision of the curriculum for secondary schools for

1. it assumed the value of traditional content.
2. it has clung to a logical organization of subject matter set-out-to-be-learned.
3. it has accepted departmentalization of learning as necessary.
4. there has been no critical evaluation of practice in terms of worthy purposes of education.

[5] J. H. Zant, Op. cit. pp. 17–18.
[6] J. H. Newlon, Op. cit. p. 298.

5. the interests and immediate needs of the pupils have been given too little consideration."[7]

He suggests a fundamental reorganization of the secondary school curriculum which will lead to larger units than we now have, each focused on one of the objectives of secondary education. Each of these objectives implies attitudes and skills of importance in the best social living, and each of these attitudes and skills in turn is "underlaid by a whole hierarchy of smaller but similar and supporting understandings and skills." Both pupils and teacher would understand that the subject matter is a means, and that it would be worthless to gain the subject matter without achieving the objective. This may be brought about by making sure that the unit test is in terms of "understanding of, attitudes toward, or skills in the objectives of the unit, not in the subject matter of the unit."

"These units when built should be assigned to the department and teacher who, by reason of training or experience, has a good command of, respect for, or interest in the objective being stressed by the unit. Usually this will mean one who is in command of most of the subject matter involved. There should be no omissions of subject matter, however, to fit into departments of teacher knowledge . . . To make up for any deficiency in teachers' training, a teachers' handbook to accompany the unit should be prepared."[8]

Many people, including those interested in general education as well as those interested in some particular subject, would not agree with all the ideas expressed by Dr. French. For example, advanced practice in the teaching of mathematics does not assume "traditional value of subject matter." Nor will specialists in this field agree that "there has been no critical evaluation of practice in terms of worthy purposes of education." Dr. French makes the common mistake of choosing the worst practice in the older type of curriculum and comparing it with the best in the new.

His method of dealing with the subject matter needed in the development of the units is also weak. To take the ordinary teacher of social science or of any other subject and hope to make up his

[7] Will French, "Newer Practices in High School Curriculum." *Curriculum Making in Current Practice*. School of Education, Northwestern University. 1933. pp. 134–135.

[8] Will French, Op. cit. p. 135.

deficiency in an unfamiliar field, like mathematics, by the use of a teachers' handbook is to overlook the importance of scholarship together.

The second proposal is included in the tentative bulletin, "Scope and Sequence of the Curriculum for Virginia Secondary Schools," which recommends for each year of the four-year high school eight or more *Functional Phases of Social Life* with corresponding centers of interest. The suggested units of work for each big topic may lead to the use of the subject matter from the various fields. During the first year it is suggested that the subject matter from all the usual fields be used at various times, though it would be possible for the teacher and pupils to select units that would exclude some particular subject for the entire year. For the other three years of the high school the suggested subject matter for the unit does not include any mathematics and very little from the fields other than from social science and English.

The third proposal is by Dean Stout of Northwestern University. It seem much more likely to succeed than the others that have been discussed. He says:

> A movement is now in prospect in the secondary schools to attempt the organization of at least a few *functional* units which will ignore subject and even field limitations. . . . The teaching and learning of subjects and fields as such still dominates educational practice in spite of all that has been attempted to secure revision and reorganization of curricula.
>
> Organization of one such unit or even two corresponding in both name and purpose to each of the four objectives of secondary education (health, leisure, vocation, and social relationships) will permit continuance of one or more subject units in each of the traditional fields of instruction. The former will provide the common elements in the curricula and furnish the means for a liberal or general education while the latter may serve in supplying, in part at least, the differentiated ones required for college entrance and later specialization in the traditional subjects and fields.[9]

If the integrated curriculum is to be acceptable to secondary teachers and administrators and receive the backing of college teachers, it will have to be worked out by people who are interested in the different subjects, and who are familiar enough with the subject matter to be able to see the relations of the various parts. Ideally a person developing or teaching such a unit should be familiar with much of all the subjects involved. To be able to see relationships properly, he should know much more about the sub-

[9] J. E. Stout, "Changing Concepts of Units," *Curriculum Making in Current Practice.* School of Education, Northwestern University, 1931. pp. 114–115.

jects than any teacher would expect to teach. In high school teaching it would mean that one would almost have to specialize in many different fields, such as English, mathematics, art, social science, and the like.

These proposals in regard to the complete reorganization of the schools in regard to the content and to the methods of teaching are not to be dismissed lightly. The idea has many good features and seems to be taking a strong popular hold with certain types of teachers and administrators. The sponsors of these programs are among the most highly respected educational leaders of today. These leaders are also working on their proposals with an almost religious zeal.

The danger is in leaving the detailed phase of the reorganization to men whose vision may be limited and thus getting a curriculum that is weighted in some particular direction. There are indications that this has been done in some of the courses of study now in use and in some of the proposals set forth. Specifically, the teacher of mathematics cannot adopt a "hands off" attitude merely because he does not agree with some of the things that have been done. The interests of the pupils should be placed first.

The chief point of danger is probably in the secondary field. This is especially true because college teachers expect entering students to have definite skills and knowledge. If college teachers are going to make these demands, they should also be able to make constructive suggestions as to the organization of learning in the high schools. To be able to do this one must have a broad knowledge of, and a tolerant sympathy for, various programs of reorganization.

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## Notice to Subscribers

Blanks to be used in getting subscribers to *The Mathematics Teacher* will be sent postpaid upon request. Please indicate the number needed.

# Alice in Dozenland

*By* WILIMINA E. PITCHER
*Cleveland, Ohio*

SCENE I. Guard post in Rawlings School.
SCENE II. School room in Dozenland.
SCENE III. Guard post in Rawlings School.

*Characters*

| | |
|---|---|
| ALICE | NUMBERS |
| TEACHER | SYMBOLS |
| GUARD | CHORUS |
| TWO RAWLINGS BOYS | |

Numbers and symbols should be small children. Each has a placard telling what he represents suspended from his neck. There should be $+$, $-$, $\times$, $\div$, $=$ and the Dozenland digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ↑, ∠. There should be two 1's. Any other numbers may be duplicated. Numbers must be able to walk without bending knees, and to stand quietly. The more stiff and mechanical they appear, the better the effect. Numbers may move at sound of their names, when pointed to, or when helped. Numbers 6 and 4 are broken and must be moved by guard. They come in with others as all are helped by guard.

CHORUS: Between scenes, the chorus sings twice the chorus of "I Can't Do That Sum," from Babes in Toyland.

*Properties*

Two extra fingers each for TEACHER, ALICE and GUARD in SCENE II.

Placards with following problems:

| | |
|---|---|
| $6+8=12$ | $4\times7=24$ |
| $7+8=13$ | $16\div2=9$ |

Placards with Dozenland multiplication tables. These are correct.

| | |
|---|---|
| $1\times5=5$ | $1\times3=3$ |
| $2\times5=$ ↑ | $2\times3=6$ |
| $3\times5=13$ | $3\times3=9$ |

| | |
|---|---|
| $4\times5=18$ | $4\times3=10$ |
| $5\times5=21$ | $5\times3=13$ |
| $6\times5=26$ | $6\times3=16$ |
| $7\times5=2\angle$ | $7\times3=19$ |
| $8\times5=34$ | $8\times3=20$ |
| $9\times5=39$ | $9\times3=23$ |
| $\uparrow\times5=42$ | $\uparrow\times3=26$ |
| $\angle\times5=47$ | $\angle\times3=29$ |
| $10\times5=50$ | $10\times3=30$ |

## ALICE IN DOZENLAND

SCENE I. Guard post in Rawlings School.

*When curtain rises,* ALICE *is seated in a chair in the school corridor, trying to solve an arithmetic problem.*

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ALICE: (*Throwing paper on floor.*) Oh, how I hate that math! (*rising and walking to front of stage*). Every problem wrong yesterday, every problem wrong today, and, I suppose, every problem wrong tomorrow, and all on account of a nice little dot called a decimal point. And I honestly believe that half the math teachers in Rawlings are crazy about the things. You'd think they were, to hear them talk. You should have heard our teacher today when I said I hated the way we write numbers. Wasn't she shocked! She said, "Why Alice, we have a very beautiful, convenient number system." She also said something about decimal meaning ten, and that our entire number system, both whole numbers and decimals, used tens. I asked her why they picked tens. She said "Probably because people first counted on their fingers, and we had ten fingers." (*Returning to her seat.*) Well, if that's why, I most certainly wish I did not have ten fingers. Now, let's see. How many would I want? (*Trying out various numbers.*) Two? No, my hands wouldn't be very useful. Five? That looks funny. So sort of uneven. Six? Looks like birds' claws. Maybe I need more than ten. (*Looks admiringly at her hands which are bedecked with rings.*) I'd have room for an-

other ring. (*Still looking at hands mutters drowsily*). 8, 12, 4, 6, 12. (*Sleeps.*)

CURTAIN

CHORUS:

SCENE II. Schoolroom in Dozenland.

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ALICE: (*Entering, looks around bewildered.*) Well, this is a queer looking schoolroom. How did I get here?

GUARD: (*Entering from other side of stage.*) You wished yourself here.

ALICE: No, I didn't. I——

GUARD: Don't contradict me. You most certainly did or you wouldn't be here.

ALICE: That's funny. I can't remember wishing myself here. Now let's see. The last thing I remember I was hating math. Nothing unusual about that. I always hate math. Then I was hating decimals. I still do. Then I wished I didn't have ten fingers.

GUARD: And you got your wish, and that's why you are here.

ALICE: (*Holds up her hands. Each hand has six fingers.*) Isn't that funny! I have twelve fingers!

GUARD: No, you haven't.

ALICE: Why yes, I have. But I don't think they improve the looks of my hands one bit. And I can't see that this one or this one will be very useful.

GUARD: Fine for counting.

ALICE: Why of course. And perhaps I did wish myself here when I was hating decimals. Now let's see how these numbers go. (*Counts.*) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (GUARD *nods approval at each* NUMBER.) 12.

GUARD: No!

ALICE: What?

GUARD: A big girl like you and can't even count your own fingers.

ALICE: I can too count. Why I'm in 7A at Rawlings, and I know all about decimals, and—(*loud laughter off stage.* ALICE *looks around bewildered.*) Well, well, we have decimals in 7A at Rawlings.

GUARD: (*Groans*) Ah, gee! Now I'll have to go find her a math

teacher, and I'll have to get all those numbers out again and dust them off. I'll bet I'm going to have an awful time. (*Goes out grumbling.*)

ALICE: (*Looks about. Reads math cards on the wall.*) Six plus eight equals twelve. Seven plus eight equals 13. Four times seven equals twenty-four. Sixteen divided by two equals nine. Well, at least I know better than that.

GUARD: (*Returning, still grumbling.*) No, you don't know better than that. You don't know better than anything. In fact, you don't know anything at all. I found you a teacher. Won't she make you work if you stay here! And if you don't work, you'll lose two fingers and have to go back to Rawlings and your old decimals. Wouldn't you hate that! Here's your teacher now, so I'll go and get those pesky numbers. (*Exit.*)

TEACHER: Good morning. I am Miss Smith. I am to be your mathematics teacher. We are very glad to have you in our school. What is your name?

ALICE: Alice, Alice, well maybe it's Rawlings.

TEACHER: How old are you?

ALICE: Almost 13.

TEACHER: I didn't understand.

ALICE: Almost 13.

TEACHER: Thirteen? There is no number 13.

ALICE: Oh yes. You know 10, 11, 12, 13.

TEACHER: Oh my dear child. The third number after 10 is one dozen and one.

ALICE: Oh dear. Maybe this is going to be worse than decimals. Of course, I know what a dozen and one is, but I've always called it 13, and——

TEACHER: But you wished all that away. You must use dozens now. The first thing you must do is learn to count.

ALICE: I can count. I could count millions if I had to.

TEACHER: There are no millions in Dozenland. The guard said you could not count your own fingers. Let me see how far you can repeat numbers.

ALICE: (*Rapidly.*) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,——

TEACHER: Wait, Alice, what comes after 11?

ALICE: Twelve.

TEACHER: No,—one dozen.

ALICE: Yes. Twelve is one dozen, but we don't call it that in Rawlings.

TEACHER: You must call it dozen if you stay here. Now go on. Eleven, one dozen.

ALICE: Eleven, one dozen, thirteen.

TEACHER: No, no. One dozen, one dozen and one, one dozen and two. Go on.

ALICE: (*Doubtfully.*) One dozen and three, one dozen and four. Aren't there any teens?

TEACHER: Not in Dozenland.

ALICE: How do I get to 20?

TEACHER: You left 20 behind when you wished yourself out of Rawlings. Go on. Start with one dozen and eight, one dozen and nine. Go on.

ALICE: One dozen and ten, one dozen and eleven, one dozen and twelve.

TEACHER: No, no. Not one dozen and twelve, but two dozen.

ALICE: Oh! And I suppose it goes two dozen and one, two dozen and two, up to three dozen, and then starts over again, three dozen and one.

TEACHER: Yes, you have the idea. You will soon be able to count well.

ALICE: I was always good with numbers. (NUMBERS *off stage groan loudly.* ALICE *looks about surprised and indignant.*)

GUARD: (*Stepping into room.*) Miss Smith, I have the numbers.

TEACHER: Bring them in, please.

(*The* NUMBERS, *all holding to a heavy rope, are pulled in by the* GUARD. *When they are in, the* GUARD *passes down the line, taking the rope from the hand of each* NUMBER. *When the rope is released, the* NUMBER *turns to face audience. 4 and 6 do not turn. In the meantime,* ALICE *is walking about examining the* NUMBERS *in a puzzled fashion. Occasionally she goes to* MISS SMITH *and whispers a question.*)

GUARD: (Turning 6 and 4.) 6 and 4 are broken as usual.

TEACHER: (*To* GUARD.) Thank you. (*Turning to* ALICE.) Now you must learn to form numbers.

ALICE: I was always good with numbers. (NUMBERS *groan.* ALICE *looks surprised and indignant.*)

TEACHER: Are you acquainted with all the numbers to one dozen?

ALICE: (*Looking at them, and pointing.*) Oh, yes, I know them. There is 1, 2, 3, 4, but some one has mixed those funny looking letters with them and we'll have to put them out. (*She points to* ↑ (*ten*) *and* ∠ (*eleven*) *who weep.* NUMBERS *standing next to them try to comfort them.*)

TEACHER: Oh, Alice, dear. Our numbers are quite sensitive and you must not hurt their feelings. This (*pointing to* ↑) *is ten and this* (*pointing to* ∠) *is eleven.* In the future you must be more thoughtful and courteous. Can you form other numbers?

ALICE: Well, they do look queer. Now I'd make 12 this way.

TEACHER: You mean make one dozen. You must forget that word 12.

ALICE: (*Nods.*) I'd make one dozen this way. (*Places* 1 *and* 2.)

TEACHER: But don't you see that is not one dozen. Here is dozen's place, here is unit's place. One dozen, no units. (*Places* 1 *and* 0.)

ALICE: But that is 10.

TEACHER: No, Alice. There is ten. (*Points to* ↑.) Your number is one dozen and two which is four more than 10.

ALICE: How silly. I suppose, then they would make 13, one dozen and one, this way (11), 15 this way (13), and 20— Let's see that would be (*counts on fingers*) one dozen and eight. They'd make it this way (18). Oh dear, it's sort of like a puzzle, but maybe I can learn it. I was always good at—(NUMBERS *groan.* ALICE *looks indignant.*)

TEACHER: Of course, you can. You will also have to learn to add, subtract, multiply, and divide with dozens. I am going to give you a few simple addition problems. (TEACHER *places* 9+6= , ↑+4= , 8+6= . *She gives* ALICE *time to solve each problem, and* NUMBERS *time to return to their places before placing next problem.* + *and* = *remain in place between problems.*)

ALICE: 9+6. Now that's 15. (*Counts on fingers.*) 13, 14, 15, 1 dozen and 3. (*Places* 1, 3.) 10+4 (*Counts on fingers*) that's one dozen, two (*places* 1, 2.) 8+9. Oh dear. That's hard enough in Rawlings. I think it is one dozen and five, (*places* 1, 5). Well, then, (*pointing to wall cards*) those are right, after all. 6+8 equals one dozen and 2; 7+8=one dozen and three; 4×7=two dozen and 4;

16, I mean one dozen and 6, divided by 2 equals 9. Well, maybe addition wouldn't be so bad, but I'm not so sure about the rest of it. I'd have to learn a lot of new multiplication tables like those. (*Points.*) If I learn subtraction, I'll have to learn borrowing. I suppose if I borrowed from the dozens column, I'd have to borrow one dozen. That would be much worse than tens. And suppose I had to do long division with dozens. Oh, that would be awful! I wish I didn't have twelve fingers. I wish I were back in Rawlings. (NUMBERS *rush from stage.*)

CURTAIN

CHORUS:

SCENE III. Guard post in Rawlings School.

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ALICE: (*Awakens and looks at her hands.*) Then it isn't true. I haven't a dozen fingers. I couldn't have been in Dozenland. It must have been a dream. It was quite an interesting dream, and I liked that teacher. I do wish I had remembered to ask her what comes after eleven dozen and eleven. How I hate that guard. He was such a smarty. No guard in Rawlings would ever dare be as rude as he was. (*Spies two Rawlings boys in distance. Starts indignantly after them.*) Hey, you guys. Who do you think you are, anyway? Got a pass?

CURTAIN

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# What are the Characteristics of the Progressive Mathematics Teacher?*

*By* ELIZABETH DICE
*North Dallas High School, Dallas, Texas*

I SHALL TALK of three general characteristics of the progressive mathematics teacher: first, scholarship; second, individual characteristics; and, third, capacity for growth. The third characteristic, capacity for growth, is my tribute to the word *progressive* in the topic assigned to me.

I have subdivided the first characteristic, scholarship, into two parts: mathematical training and teacher training. When someone spoke of Dr. E. R. Hedrick of the University of California as not only a mathematician but also a teacher, I though of the Irishman's two-men-in-the-same-grave story: "Here lies a lawyer and an honest man!" In college, however, when I saw the department of mathematics and the department of education at each other's throats, I could understand the distinction. Mathematicians, scorning the plans of the school of education, insisted that mathematics exists and can be learned or taught by born mathematicians. Why should anyone who *knows* bother with *plans?* They scorned directions and even a reasonable course! An address (I saw it in mimeograph form; I do not think it is in print) made by Miss Marie Gugle, assistant-superintendent of Columbus, Ohio, before one of the northeast conferences for teachers stresses our unreasonable expectations in mathematics. Miss Gugle, a mathematician, a teacher, an author, and a superintendent, should know. Mathematics is a language, a language of symbols; yet we expect more of children in this than we do of adults in other languages. There are adults who require years to perfect an accent in other languages.

Members of the school of education went to the other extreme. Of course, they, with a magic plan book, could teach mathematics. They felt indignant towards the professor of mathematics who would not recommend the prospective coach as an instructor in mathematics. The coach and would-be mathematics instructor

* Given at the Fourth Annual Teacher-Training Conference of North Texas State Teachers College at Denton, Texas, March 10, 1934.

had had no college courses in mathematics. Well, why couldn't he take a six weeks' course in the summer? Yes, why? Years to perfect an accent in other languages, but the language of mathematics in six weeks!

Why all this dissension? Schools of education sprang up overnight. The experimental stage was a necessity. No intelligent experiments, no growth. Schools of mathematics, with the dignity of age, had no patience with the new. Time went on. Teacher-training gained strength. Mathematicians, helpless before the onslaught of the non-born mathematicians, began to feel the need of this strength. Thus, the two extremes began to swing towards each other. Increasing harmony between the two is one phase of educating for the new age, the general theme of this conference. There is no royal road to geometry, but with common-sense maps and with a trained guide, the journey can be made an enjoyable one.

*Mathematical training*. Dr. W. D. Reeve of Columbia University, in the fourth yearbook of the National Council of Teachers of Mathematics, suggests that "through calculus" should presently be required for teachers of high school mathematics. "Presently be required" was well put in 1929, the year the fourth yearbook was published, because high schools had grown so rapidly that there had been little time to make definite requirements. In *The Mathematics Teacher*, March, 1934, however, Dr. Reeve says we are no nearer our goal than we were in 1929, five years ago. He declares that mathematics would not be difficult if properly taught, and censures the administrative officers for not trying to employ teachers who have majored in the subjects they are to have.

Mr. Edmondson of Dallas in October, 1932, *The Texas Outlook*, gives a definite list of mathematical requirements compiled by the late Dr. J. W. Young. This list is much more elaborate than Dr. Reeve's "through calculus" but, in the main, could be so summarized. It is understood that trigonometry, college algebra, analytic geometry, or their equivalents, precede differential and integral calculus. Practically two years of college mathematics would be included. Contrast such an excellent beginning with the present conditions: Only one-third of the Texas mathematics teachers have majored or minored in mathematics. This means that not one-third of the Texas teachers of mathematics have majored in mathematics, and two-thirds of them have not even minored in it. More

than likely, these two-thirds took the minimum requirements in both high school and college. Generally speaking, other states are not far in advance of Texas.

In addition to "through calculus," the progressive mathematics teacher should know, or should have in his library for frequent reference, the history of mathematics, interesting bits from the lives of mathematicians, mathematical magazines, new books, and the yearbooks of the National Council. Also, he should attend conferences, mathematical programs, etc. Such a background not only enriches a course but also inspires confidence. The psychology of knowing higher mathematics cannot be overestimated. It has been said that a horse knows when its rider is afraid. We all believe that dogs recognize the people whom they can trust. Certainly children are inspired by him "who *knows* and *knows* that he *knows*." You remember the story of the old professor who was asked by his young instructor, "What do you do when students ask you how so and so is used?" And he gruffly replied, "I tell them." The Sixth Yearbook of the National Council gives many such uses of mathematics.

I do not mean to leave the impression that the progressive mathematics teacher should know "more and more about less and less." Combination teaching positions exist and should be prepared for. Teachers with full time mathematics programs should constantly be on the alert for allied interests. Helpful correlations should be made. This calls for a knowledge of more than one field, a reference knowledge, an appreciation of a number of fields, while specializing in one or two. Dr. Bagley warns against too little knowledge for working purposes. "*Confusion* rather than *fusion* courses" is the way he puts it. Josh Billings says, "It is better not to know so much than to know so much that ain't so." I am not talking of this extreme, I am talking of the need for scholarly minds in teachers and parents, the need for thoroughness and for sound foundations. Think of the mischief done by poor arithmetic teachers! The children are given a distaste for numbers and are not given the fundamental elements of mathematical thinking. Perhaps less mischief is done by poor calculus teachers. At least they work with less plastic minds. Miss Decherd of the University of Texas asks if Dr. Judd, when he declares arithmetic is poorly taught, is a pessimist or a wise man. Mr. Miller, superintendent at Eagle Pass, asks these significant questions: "Who will see about

the poor arithmetic teacher? The school of education?" Mathematicians deserve his thrust. Arithmetic teachers, high school teachers, college mathematicians, and representatives from the school of education should work together "seeing about" poor teachers, poor methods, poor courses of study, etc. A Dr. Hedrick, a mathematician and a teacher, should, as a balance wheel, be one of the group. Administrators, board members, and the public should at least be interested in such a study.

*Teacher training.* I sincerely believe that teacher training is as important as mathematical training. Presently-to-require "through calculus" or through any course in mathematics cannot take the place of knowing how to adjust oneself to this or that school situation. Adjusting, co-operating, getting along with other teachers, with pupils, with administrative officers, with parents—in short, getting along with people—is a powerful asset.

Among the significant social requirements are: The problems of departmental co-operation; extra-curriculum co-operation; community co-operation; the carrying out the policy of the school; contributing to the effectiveness of the school; the adjusting oneself to varying programs, to revised courses of study, to new demands in counseling and in guidance; the making time for seemingly unhurried conferences; the learning to profit by yesterday's errors; and the study of how children think and thinking on their level.

I like to tell this story: One day in plane geometry, we, as a class, were studying an original, thinking, suggesting, experimenting, discarding, thinking again. Finally, a serious little boy, his face alight with eagerness, leaned forward to ask, "Can *you* do it?" Just as seriously, I replied, "I can when I am your teacher, but to-day we are thinking together. It is a game we are playing, and I *have* to be fair." "Oh, no'm, you can't think ahead of us" was his understanding response.

Child psychology is a sympathetic understanding of and abiding interest in young people. "Anticipating difficulties and forestalling them" is an item in a teacher's creed in *The Mathematics Teacher* for March, 1926. "Feeding and encouraging the sense of wholesome curiosity," is another item. Both are worth repeating:' 'Anticipating difficulties and forestalling them," and "Feeding and encouraging the sense of wholesome curiosity." All of this can be summarized in two words—provided the broadest interpretation is put on the two words—teacher training.

What are the characteristics of the progressive mathematics teacher? First, scholarship, including mathematical training and teacher training. Second, individual characteristics. Mr. Adamson, our grand *young* man of Oak Cliff, says a teacher should be pretty. Fortunately, human beings see with more than one sense. My father, from the standpoint of irregular features, was an unusually ugly man, but every irregular line in his face radiated with such kindness that I did not see his long nose or his Adam's apple. Children, even more accurately than adults, see *behind* the face; hence, I agree that teachers should be pretty.

Health is and should be the first of the seven cardinal principles of education. The wise educator practices as well as teaches the principles of health. Mr. Kimball, former superintendent of Dallas schools, when asked by the employing superintendent for advice concerning this or that applicant, invariably asked, "Has she vim, vigor, vitality?" We dubbed them the "three famous v's." Health; kindness; a sense of humor; a reasonable freedom from financial worries; a reasonable freedom from home responsibilities (worry or physical weariness colors one's judgement) are all worthy of being called cardinal principles of the progressive teacher.

The psychology of being well dressed enters into the making of a successful teacher.

Enthusiasm is a desirable characteristic. Students catch the spark! The work should be interesting. Lillian Russell declares children should be allowed to walk out of uninteresting classes. Immature minds are not always fair judges, but progressive teachers should guide them to be fair judges, and the only way to do this is to give them something to judge. There are many ingenious ways of creating interest. I like to appeal to the pupil's sense of justice. In a senior algebra class, we were taking ten minures each day for what we called "increasing the mental span." No pencils allowed. One boy said he believed he could remove the parentheses in $(2a-3b)^3-(3c+5m-r)^2$. I said I thought he could if he could keep the signs in mind and that it would be fun to try. Another boy, a sulky fellow, was looking out of the window. Later, when I had an opportunity casually to discuss his attitude with him, I told him that the particular skill was not important, but it was merely interesting to see how far we could go without a pencil, like any other game. I knew he liked golf; so I said that he should be as tolerant of our game as we were of his, for we might think it absurd for him to spend hours trying to put a small ball in a

small hole when he could go drop it in. Why put it in, anyway? I did not want him to enjoy our game; I merely wanted him to co-operate. The first boy was really enjoying removing the parentheses mentally, I was enjoying following him, and it was part of the second boy's education not to allow himself to be bored. Holding the interest and challenging co-operation, no matter how difficult, are worth striving for. Carefully thought-out plans are invaluable. Outlines and summaries should be made in class, but only the teacher who knows each step can successfully direct or can keep the students interested. A haphazard game is neither interesting nor instructive.

The language of mathematics, the power and the beauty of symbolism—Why bother with transfer of training? Study mathematics for its own sake. See "the glory of the forest!" Rapidly multiplying schemes and devices for imparting facts are, without cautious teachers, apt to be "obstructing trees." Those who do not find mathematics interesting can be taught to respect it. The farmer who boasts of not using high school mathematics does not know that solid geometry gave him the formula for finding the volume of his wheat bin. The carpenter depends on mathematics for his simplest tool, not to speak of his contractor's blue print. The doctor uses symbols in writing his prescription. Thousands of people, who either treat mathematics lightly or do not think of it at all, unknowingly use it each time they enjoy the radio, automobile, airplane, or almost any other convenience. Not in a holier-than-thou way but persistently day after day, children and their parents should be taught to respect this handmaiden of the sciences. No teacher is progressive who neglects to seize opportunities and to *make* opportunities to inculcate this respect.

Not all people can major in mathematics, but all can respect it. All can learn certain phases of it, too, if it is properly presented and if the pupils are in a teachable frame of mind. There is a point in mathematics, as in all sciences, beyond which the majority of people cannot go. That point, however, is often reached in the pupil's imagination before it is reached in his ability. Frequently parents aid the children to establish this mental hazard by boasting that *they* "couldn't learn mathematics either." Some newspaper referred to one of our candidates for governor (we need mathematics to count these candidates) as "Guillotine Tom," because he had proposed hanging kidnapers. Teachers are notably con-

servative, or I would propose hanging all "I-can't-learn-ers."

That intangible something, personality, makes or mars numerous teachers. Mental poise, tolerance, serenity and self-control in the teacher help the children to discipline themselves. Wild ideas flit through the mind, but the choosing measures logical thinking. Encourage the child to think of making sane choices. Guide him to a happier frame of mind. Tactfully point out the futility of an ugly disposition, the rightness of cooperation. Teach him to avoid I-do-not-know in an I-do-not-care tone. Talk of intelligent guessing of seeming discourtesy—all of these lessons further the development of the child. Einstein says that in studying mathematics we should never forget that creations are better than equations.

The third and last characteristic of the progressive mathematics teacher is capacity for growth. Facts today are not facts tomorrow. Changing needs, changing objectives, and changing emphasis must be considered. Yesterday we were emphasizing the formal; today we strive for naturalness. Yesterday we emphasized the how; today we reason why. We are putting common sense against tradition, training in citizenship against undue domination of colleges, and supervised study against the strangle hold of entrance examinations. A thought-out skill is retained longer and is more usable than one that is mechanically acquired. The progressive teacher, realizing these changing conditions, strives to enlarge his capacity for growth. Working for enlarged understanding, self-training, improving instruction, a desire for a progressiveness that extends beyond mere tenure of position, and an awareness of the need for supplementing his scholarship with thoughts of the new age are some of the earmarks of growth.

There are many ways to grow: (1) By teaching. Sometimes this is bad for the pupil, but it is excellent for the teacher who is trying to learn. Peck's Bad Boy trained his pups with his father on all fours, a pup snapping at each ear. The Bad Boy said it was bad on pa but it was *the making* of the pups! (2) By visiting other teachers. New ideas are everywhere if one knows how to look for them. The most successful teacher can learn from the most insignificant. The old negro backed the buggy of the worried Chief-Justice Marshall off a stump with the philosophical comment that the Jedge couldn't know everything. (3) By conferences and by lectures given by co-workers. Too often teachers think they have to "go to school" to learn. Training in service, teacher participation,

reading new material, being stimulated to re-read old material, meditating, and wanting to be a better teacher each succeeding year are other ways of enlarging one's capacity for growth. Go to school from time to time, but never forget that the most genuine learning is self-learning.

This is the picture, then, of the characteristics of the progressive mathematics teacher: (1) Scholarship with its two divisions, mathematical training and teacher training; (2) Individual characteristics including health, a sense of humor, a reasonable standard of living, inspiration, and a pleasing personality; (3) Capacity for growth. I have a frame for this picture. No picture can be properly displayed without one. This frame is indispensable. It is square. The

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four sides are T $\boxed{\phantom{xxxxxx}}$ M. In the table of contents of the February,

E

1934, "Journal of the National Education Association" is this title: "The Tragedy of the Half Principal." Being a decorous classroom teacher, instead of reading indecorous meaning into the title, I hastily turned to the article. It told of the tragedy of a principal being given twice as many buildings and teachers as he could supervise—thus making him a half principal. As the writer said, each building had an engineer but not a principal. Similarly, numerous teaching loads produce half teachers and third teachers. The public expects us to perform miracles. Given more time—more time for personal contacts, for conferences, for guidance, for study ing with small groups, for quiet thinking—we might help some of the boys and girls to perform miracles.

The frame for the picture is *Time*. The glass is *Vision*.

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# A Student-Made Slide Rule

*By* Wm. E. Bush
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For various reasons the treatment of the slide rule in high school mathematics is generally a limited and superficial affair. Many teachers do no more than mention it, while others make attempts at elucidation of the theory involved. A few go further than this by having a large classroom demonstration rule upon which both the teacher and the pupils may work while everyone but the operator watches. It is, of course, possible to furnish rules to the students, but it is an expense which may justly be called unnecessary.

Since advanced algebra in high schools is generally an elective, it seems safe to assume that a rather large percentage of the pupils enrolled are likely to be found later in college mathematics and science courses. Due to the great advantage to be found in using the slide rule in these departments, particularly in the physical sciences and engineering, it is felt that a more effective method of slide rule instruction is desirable in the preparatory subjects. Furthermore, the general usefulness of the rule is attractive to all of those who are desirous of specific examples of applied mathematical theory. It is with these ends in view that the following scheme is offered.

The general practice is to introduce the slide rule as an application of the theory of logarithms. The way in which the scales are built, once pointed out, make the operation of the rule obvious enough and the average student has no trouble in understanding the theory. But it is like knowing the theory of how the baseball pitcher throws an "incurve"; the theory is of little use to the novice without considerable supplementary experience.

One method of getting a slide rule into the hands of every student is to have every student make his own. However, if the student has to lay out the scales himself, while the experience may have some value, the result is not likely to be a very accurate or usable slide rule. Furthermore, the making of such scales is a very tedious matter and would kill the interest in most cases. The way out is simple.

Unnumbered scales are to be found available in the ordinary

semi-logarithmic paper which is printed by the makers of engineering and drafting supplies.[1] This paper consists of a ruled surface much like ordinary graph paper excepting that in one direction, instead of the lines being equidistant, they are drawn at intervals proportional to the logarithm of the linear distance from one edge of the paper. Now this is exactly how a slide rule scale is constructed; thus we may use this paper for the scales of our rule. It is a simple matter to furnish each student with two strips of semi-log paper and let him make his own rule. If this is done under the guidance of a few simple instructions, the result is a rule which is nearly as accurate as the ordinary 10 inch slide rule.

The most simple and probably the most desirable form of the rule is made in the following way. Since the paper strips cannot be handled easily, they must be mounted upon pieces of cardboard. Each strip of the semi-log paper is left about $\frac{1}{4}$ inch wide. These should be pasted upon strips of card about $\frac{3}{4}$ inch wide. This makes the scales of sufficient rigidity to be easily manipulated. One of the scales is to be the moving scale and the other is fixed. It is a good plan to paste the fixed scale on to an additional piece of card, say about an inch and a half wide. This makes it easier to place the moving scale at any desired position along the fixed portion. These two scales now constitute the C and D scales of the usual rule and are sufficient for all ordinary purposes of instruction.

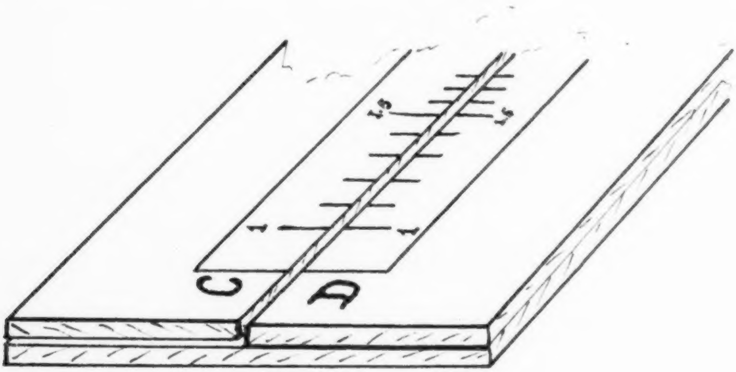


Fig. 1

[1] See Keuffel and Esser Company, New York City. Semi-logarithmic paper, 1 cycle$\times$60 divisions.

The numbers must be put on the scales and this is done in the conventional style. Single cycle semi-log paper may be used. This comes in $8\frac{1}{2}$ by 11 inch sheets with the logarithm scales in the long direction. The ruled surface is ten inches long so that this becomes the length of the rule. Fig. 1 indicates the general features of construction pointed out above.

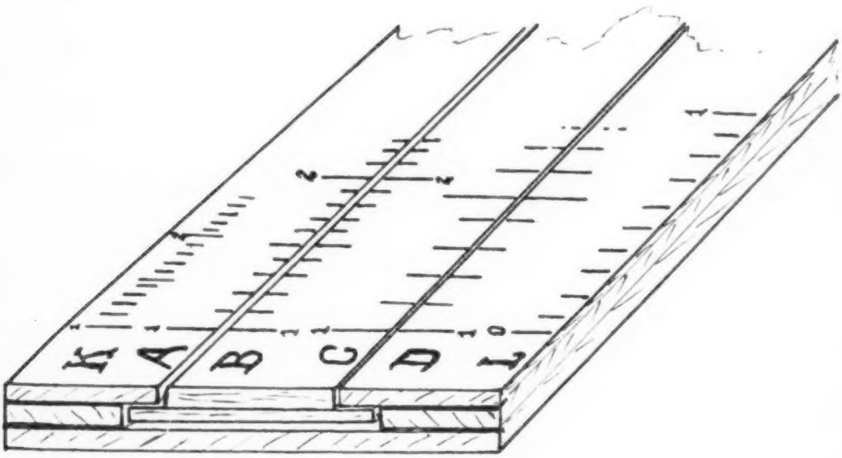


Fig. 2

Care should be taken at two points in the making of this rule. First, one must be sure to cut all the edges as straight as possible, and secondly, one must avoid stretching the strips of paper. The paper was tested in this respect and found to be quite accurate when no stretching had taken place. It was then stretched (holding only the ends) and again tested. The result showed that the paper did not stretch uniformly. In order to avoid deformation, do not put paste upon the paper strips, but only on the cardboard instead. In this way stretching is minimized. There is bound to be some stretching and care should be taken to stretch both strips exactly the same amount.

The rule, herein described, was constructed by a class of twenty-one students. The instructor furnished the paper at the expense of about eleven cents for the whole class. The scheme was greeted with great enthusiasm and a number of the students claim to be using the rule outside as well as within the class.

The general scheme may be used to a much more elaborate degree if it is so desired. The above rule contained only the C and D scales for multiplication and division. However, the writer has constructed a very satisfactory rule, having a slider, A and B scales and a log scale. The cube scale (K) might also easily be added as well as any folded scales.

The construction of this rule is the same as the other, excepting that the sliding scales are locked into place as indicated in Fig. 2. The A and B scale (square scales) are constructed from two cycle semi-log paper. A K (cube) scale can be made from three cycle paper. The logarithm scale is made of inch coordinate paper having twenty divisions per inch. The cardboard slider is easily made and an old piece of photographic film or even a piece of cellophane answers the purpose of carrying the "crosswire."

In case this more complicated rule is constructed, the following precautions should be taken. While all the scales are supposed to be ten inches long, it is found that they are not exactly so. To overcome this, paste the longest one on first and then stretch the others to exactly the same length. Also be sure that the indices all line up accurately.

When these rules are finished they are sufficiently accurate to guarantee two digits and afford a very good guess at the third. Thus not only does the construction of the rule give the student a sufficient motivating interest and act as a teaching aid but it also actually gives him a tool of immediate value in his courses in science and possibly in some of his subsequent work in the mathematics class itself.

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I CAN THINK of no other spectacle quite so impressive as the inner vision of all the manifold branches of rigorous thought seen to constitute one immense structure of autonomous doctrine reposing upon the spiritual basis of a few select ideas and, superior to the fading beauties of time and sense, shining there like a celestial city, in "the white radiance of eternity." That is the vision of mathematics that a student of its philosophy would, were it possible, present to his fellow men and women. C. J. KEYSER—*Human Worth of Rigorous Thinking*. P. 44.

# Causes for Failure in Senior High School Mathematics and Suggested Remedial Treatment

*By* Barnet Rudman
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Pupils fail in algebra and geometry because they do not apply themselves, because they have poor study habits, because they are unable or unwilling to give sustained attention, because they lack special preparation—they fail, in short, as a result of the many negative influences that militate against achievement everywhere in the high school curriculum. So, too, does inadequate teaching take its toll in the exact sciences as it does in the social sciences or foreign languages. That the percentage of failures is generally higher in mathematics than in other high school subjects is probably due not to additional specific causes but rather to the nature of mathematical skills on which the same causes are apt to leave more profound and far-reaching effects. For, the acquisition of mathematical abilities is predicated as no other learning process on sustained attention to minute details, on steady uninterrupted application, on adequate preparation, on correct habits of study and on the many other positive factors that contribute to successful learning. To teach mathematics effectively the teacher must be familiar with the psychology of learning, must be thoroughly grounded in his subject matter and well versed in the intricacies of inductive and deductive reasoning. He must be of good health, vigorous and alert to batter down the tremendous inertia that high school pupils bring with them to the mathematics classroom. Failure by pupil or teacher to meet these essentials will interfere with successful learning in any high school subject but in mathematics it often leads to catastrophe.

It follows, therefore, that the causes of pupil-failure, now quite generally known, assume a new significance in mathematics, and individually each cause requires a revaluation on the basis of its importance in this peculiar branch of learning.

Lack of native mathematical ability is often designated as a potent cause of failure in algebra and geometry. That such mathe-

matical disabilities exist in children of normal general intelligence is now believed by none save those that still cling to the discredited faculty psychology. What people call mathematical inaptitude is probably traceable to lack of adequate preparation, to a ground structure rendered unsecure through serious defects in the pupil's early training. A faulty mathematical background, then, is a major obstacle in the way of new learning, and the progressive teacher is prepared to cope with it. He does not charge the elementary school teachers with incompetence and does not evade the task of re-enforcing a shaky foundation whenever it comes to light. He may feel now and then that a pupil is a victim of careless teaching but he knows, too, that the effects of both good and bad teaching are worn off, to a great extent, through disuse, through lengthy vacations and through the multitudinous impressions that are forever assailing the mind of the modern adolescent. So he teaches the new and reteaches the old. We grow impatient when a senior interrupts our dissertation on the binomial theorem to declare that $\frac{1}{2}$ plus $\frac{1}{3}$ equals $\frac{2}{5}$, but in the end we hasten back to addition of fractions and stay with it until the erring student is thoroughly enlightened.

The weapons commonly used to combat lack of application and lack of sustained attention are not always effective; even the best of them fail to reach some of our pupils. Only the experienced versatile and well-trained teacher is adequately equipped to give battle to these enemies of mathematical learning. Understanding is the keynote to achievement in mathematics. Interest generally follows understanding, and where there is interest there is effort. No person, however well-disciplined, will continue to give mental effort to procedures that they do not understand, few will devote themselves to problems which fail to excite their interest, least of all the volatile adolescent of today.

With this in view the competent teacher strives to promote understanding by using developmental methods well within the reach of his pupil's intelligence, by frequent reteaching and clarification of obscure points and by searching for cases of lack of understanding through diagnostic testing. He stimulates interest by making his subject matter vital and relevant to the pupil's everyday life. Some of us are even capable of sufficient inspirational teaching to lighten in our pupils a spark of enthusiasm for the eternal truths of mathematics. Once understanding and interest

have been secured, the sensible teacher does not dissipate them by prohibitive assignments, or assignments carelessly made and poorly motivated. He makes his objectives clear, and attainable, and pleasurable to the pupil when attained. Even then he fails with the habitual sluggard and the pupil physiologically incapable of sustained attention, but with the majority of his class the victory is won.

Inability to study or the wrong approach to problem solving is another arch-enemy of mathematical achievement and once recognized teachers of mathematics are waging incessant war upon it. The entire classroom procedure, in fact, is calculated to develop in pupils those analytic powers essential to successful problem solving—in geometry through drill in analysis, in algebra through inductive and deductive reasoning, Skillful teachers are cultivating correct habits of mathematical thinking in every question they ask and in every technique they develop. Informal remedial work of this sort is always in progress and, if re-enforced occasionally by a period of supervised study, it is generally sufficient to counteract the evils of incorrect study habits.

All other causes of failure are secondary in nature. Either they are derivatives of the primary causes already described or they are fostered by the teacher's inability to set the right standards or his lack of energy to enforce them. Refusal to assume responsibility, careless written work, failure to make up work lost through absence, and the like are minor problems which, barring exceptional cases, energetic teachers can solve with the aid of suitable standards.

Such are the problems confronting the teacher that would cope with the plague of failure. His best efforts often fail to bring him to his goal. He cannot succeed in algebra where his pupils have no foundational mathematics at all. He cannot teach geometry where his pupils completely lack the faculty to reason. He cannot teach any mathematics where his pupils do not attend classes regularly. In attempting to stimulate interest he cannot compete with the movie, the automobile, the radio and the many other forces that thrill and stir the modern pupil beyond the capacity of sheer intellect to resist them. But within certain limitations the professionally-minded teacher, the teacher with a sense of responsibility to his work, can and does save annually many a pupil from the injurious effects of failure.

# Carl Friedrich Gauss

*Born at Brunswick, April 30, 1777*
*Died in Göttingen, February 23, 1855*

At an early age, Gauss showed unusual ability in mathematics. In fact, some say that he was only three when he corrected his father's calculations of the pay due men working under him. The father was a brick layer and Gauss was brought up in circumstances that barely escaped poverty. At the elementary school in Brunswick, he attracted the attention of an assistant teacher Bartels who was later to become professor of mathematics at Kasan in Russia and then at Dorpat in Germany. At Brunswick, Bartels' duties included cutting quill pens for the younger boys and helping them with their writing. Bartels read mathematics with Gauss and introduced him to the binomial theorem and to infinite series when Gauss was only twelve. Gauss attended the gymnasium in Brunswick and in 1792 with the financial support of the duke of Brunswick who had become interested in him, he went to Caroline College in Brunswick and later to Göttingen. At Caroline College Gauss worked in mathematics and in languages. When he entered the university in 1795, he had made progress in the theory of least squares, but he was still undecided whether to work in philology or in mathematics. His career to this point has been called his "prehistoric period." During this time, his study in the theory of numbers was largely experimental—the assembling of many cases, the forming of a rule, the proving of the theorem.

In March, 1796, he discovered a Euclidean method of inscribing a regular polygon of seventeen sides in a circle. The statement of this is the first entry in a notebook in which he listed his discoveries in mathematics. It was his first paper to be published, and it seems to have been the thing which made him choose mathematics as his field.

He left Göttingen in 1798 to become a private tutor in Brunswick. The duke of Hanover continued his stipend and Gauss was chiefly occupied in writing his *Disquitiones Arithmeticae* dedicated to his patron and published in 1801. In 1799, he published his doctor's dissertation which contained the first of his several proofs of the theorem,—every integral algebraic function in one variable,

can be expressed as a product of real linear or quadratic factors. His degree was granted by the University of Helmstadt in Hanover where Pfaff was professor of mathematics.

In 1801, Gauss became especially interested in astronomy. Bode (1778) had noticed that the distances of the planets from the sun are nearly in the ratio $0+4, 3+4, 6+4, 12+4, \cdots$, with an extra term occurring between the distances for Mars and Jupiter. An astronomer in Italy located the first of the asteroids which occupy this vacant place in the series, and Gauss calculated its orbit. For the next twenty years, Gauss was chiefly interested in astronomy.

In 1803, Gauss was invited to go to St. Petersburg, but the duke of Hanover refused to let him go.

In 1806, the duke was made commander in chief of the Prussian forces moving against Napoleon. The duke was wounded at Auerstadt and died shortly after. Gauss was then left without a patron. He refused another offer from St. Petersburg and accepted the post of director of the observatory at Göttingen and professor of astronomy there. The reputation which Gauss then enjoyed and the desire that his contemporaries had that his work should not be interrupted is shown in a circumstance arising from the French occupation of Göttingen (1807–1810). Gauss was required to make a war contribution of upwards of 2000 francs. This was excessive in view of his salary at Göttingen and his friends doubted his ability to pay it. The astronomer Olbers in Bremen promptly sent Gauss the needed amount which Gauss promptly returned. Laplace paid the money into the Treasury in Paris, but Gauss proudly sent him both principal and interest. He refused to ingratiate himself with Napoleon who might have been greatly interested in him, and somehow met his assessment without outside aid.

The work at Göttingen suited Gauss's temperament. His lectures were clear and lucid. He refused to allow his students to take notes during them lest they should lose the thread of the argument. His motto for his written work was "Pauca sed matura."

A contemporary, von Walterhausen, spoke of him in these terms,

If Gauss had wished to carry out ambitious (non-scholarly) plans in life, he would have been able to do so by means of his genius. But he had never stretched his hands even for the honours which people brought to his doors; he remained up to his last days, as he was in his youth and in his hoary years, the simple Gauss. A small study room, a small working table with a green cover, a writing desk (for standing at) painted with white oil colour, a narrow sofa, and an arm chair after his seventieth

year, a single dark-burning light, a bed-room that could not be heated, simple food, a sleeping coat and a silk cap, these were all his necessities.*

It is said that Gauss left Göttingen only twice after he undertook the work there. Once was to visit von Humboldt in Berlin in 1828 and the other occasion was when a railroad was first opened to Göttingen in 1854.

Ball summarizes his work in these terms,

> . . . the ground covered by Gauss's researches was extraordinarily wide, and it may be added that in many cases, his investigations served to initiate new lines of work. He was, however, the last of the great mathematicians whose interests were nearly universal; since his time the literature of most branches of mathematics has grown so fast that mathematicians have been forced to specialize in some particular department or departments.†

His character can be gussed by the affectionate way in which his friends referred to him as "our Gauss."

VERA SANFORD

* Ganesh Prasad, *Some Great Mathematicians of the Nineteenth Century*, Benares, 1933, p. 66.

† W. W. R. Ball, *Short Account of the History of Mathematics*, 1915 ed., p. 451.

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# Abstracts of Recent Articles on Mathematical Topics in Other Periodicals*

*By* NATHAN LAZAR

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### Algebra

1. Collins, Joseph V. *Should algebra and geometry be made elective in the high school?* Wisconsin Journal of Education. 66: 418–19. May 1934.

An enthusiastic analysis of the important rôle that mathematics has played in the intellectual history of mankind. The author regrets the diminished time devoted to its study in the elementary school and high school. He is however opposed to the demathematization of algebra and geometry that is commonly practiced to meet the needs of the great mass of students. He proposes instead that school authorities should "permit pupils who show lack of interest or ability and who are a drag on the rest of the class to be transferred to other subjects, say at the end of six or nine weeks."

But the author has failed to reckon with the teachers of the other subjects! For, unfortunately they too have discovered that the same pupils lack interest and ability in their subject and would like to have them transferred, say, from Latin, French or Physics to an easier subject like algebra or geometry.

2. Goodrich, M. T. *Concrete Interpretation of directed numbers*. School Science and Mathematics. 34: 623–35. June 1934.

"The specific objectives are: (1) to describe some concrete illustrations of directed numbers both real and imaginary; (2) to explain some concrete interpretations of the use of directed numbers in the fundamental processes; and (3) to outline a method of teaching which has been employed successfully."

3. Lee J. M. and Hughes W. H. *Predicting Success in Algebra and Geometry*. School Review. 42: 188–96. March 1934.

The purpose of the experiment was to determine the relative values of various factors for the prediction of the first semester success in algebra and geometry. The subjects of the experiment were the pupils in three Junior High Schools 213 in algebra and 125 in geometry. "The results show clearly that the aptitude tests give the best single prediction of achievement as measured by standardized tests both in algebra and in geometry. The intelligence tests provide the second best prediction of achievement." There is however, a higher predictive value to the test of algebraic (or geometric) ability when it is combined either with the score obtained on the trait rating scale or with the intelli-

* This is the first of a series of abstracts of recent articles on mathematics in other periodicals than *The Mathematics Teacher*. Other abstracts will appear in subsequent issues. This department will hereafter attempt to index and to summarize as many articles as space and time will permit.—THE EDITOR.

gence quotient. Critical scores are discussed and implications for guidance are made.

4. Richter, Rose. *The Predictive Value of I.Q.'s for Success in Algebra.* High Points. (Board of Education, New York City), vol. 16, no. 8, pp. 45–48. October 1934.

The writer reports a study on 161 cases all taking algebra for the first time. The I.Q.'s as measured by the Terman Group Test of Mental Ability, ranged from 77 to 147. The median was 103. The mental age showed a range from 12–0 to 19–6, the median being 15–0. The term's averages varied from 20% to 98% with a median of 69%.

All correlations were computed by the Pearson Product-Moment method and were as follows:

a. The correlation between the I.Q.'s and the term averages was .30 (P.E. = .05).
b. The correlation between the mental ages and the term averages was .26 (P.E. = .05).

Other conclusions based on data included in the report were:

a. There was little difference between the work of the lower I.Q. group and that of the higher one as measured by the term averages.
b. A rather large percent of the students in each of the lower I.Q. groups passed the term's work.
c. The higher and lower groups shared about equally in the exemptions from the final examinations.
d. Although the I.Q.'s and the M.A.'s of the boys and girls were about the same, only 48% of the boys made term averages of 65% or over, whereas 83% of the girls made such marks. Only 46% of the boys with I.Q.'s below 100 passed, whereas 67% of the girls with similar I.Q.'s passed.

The parting comments of the writer are "that since the study is based on first term algebra only and on a rather small number of cases, it is not conclusive. However, the results indicate that though there is some correlation, it is too low to have predictive value. Omitting the extreme cases we get a very definite indication that the lower I.Q.'s do about as well as the higher ones. A low correlation may indicate that we are overemphasizing the amount of intelligence required to learn algebra."

Unfortunately the validity of the conclusions is vitiated by the fact that the algebraic accomplishment of the students was not measured by a standard test, but rather by the term averages based on teachers' marks which are notoriously unreliable. "Passing," that is, being permitted to go on with the next grade of the subject, cannot be seriously considered as an objective criterion of success. For, as every teacher knows, it is so often motivated by mistaken kindness and unjustified hope.

5. Urbancek, Joseph J. *Typical Divisions of Ninth-Year Algebra.* School Science and Mathematics. 34: 743–51. October 1934.

"This paper is a report of the findings in a recent study made by the writer and suggests some trends in our modern ninth year algebra course. Evidence is presented indicating the extent to which general divisions constitute an elementary course in algebra and the nature of the changes that have been effected within recent years."

In the preparation of the above study, fourteen textbooks were examined and more than one-eighth million exercises and problems were counted, classified and recorded.

Of the twenty-eight conclusions drawn by the author the following are of special

interest. They afford ample proof that even that impregnable stronghold of tradition—the mathematics textbook—must yield eventually to the constant bombardment of progressive ideas and teachers:

a. Textbook writers of ninth year algebra are in agreement on the general divisions of mathematics.
b. There are more exercises and problems in the division termed Equations, than in any other.
c. The percentage of equations in a course in ninth year algebra has remained constant for the last fifteen years.
d. Textbook writers show a definite tendency to offer less formal exercises in factoring than previously existed.
e. There is less tendency to treat graphs as a separate topic, and a marked tendency to teach them in connection with equations, formulas, variation, and dependence.
f. The trend in the use of parentheses is to avoid elaborate exercises and to stress their functions while teaching manipulations.
g. Formal exercises on special products have decreased more than fifty percent in the past fifteen years.
h. There is a tendency among the writers of textbooks to include a division called Supplementary Material.
i. Trigonometry is definitely a part of the ninth year algebra course. Exercises using the tangent function occur more frequently than do those of sine and cosine.
j. Stressing of fundamental operations and using the negative sign as mere formal exercises are getting less support than in the past. Applications of the principles are on the increase.

**Arithmetic**

1. Cook, H. M. *Broadening the Basis of Study in Arithmetic*. The Mathematical Gazette (London, England). 18: 192–94. July 1934.

The word arithmetic really covers two subjects: "the pure arithmetic of the study of number which is one of the bases of all mathematical work (another being the study of space, with geometry as its primary form); and, the application of arithmetic to various customs in the civilized, and particularly the financial world, and to calculations in connection with scientific measurement." The number concept "is one of the highest if not the highest development of the human power of thought, but this development has been achieved little by little through continual interaction with the concrete, and the child, like the race, must acquire power over number by using it in daily life—in counting and measuring the objects around him in school and home—in computing costs and making payments—in reading numerical statements—in visualizing larger and larger quantities—and smaller and smaller quantities too. No custom which involves the use of number should be despised by the teacher who has himself any conception of the abstract study of number; but the teacher and pupil who by nature prefer the abstract are comparatively rare—the majority want to know the practical use to which every type of calculation can be put, and it is in the application of number that they recognize a human interest."

**Geometry**

1. Bennecke, F. *n-Teilung Beliebiger Winkel für alle rationalen Zahlen n.* (The division of any given angle into a rational number of equal parts.) Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht

aller Schulgattungen. (Leipzig.) 65: 274–79. 1934, no. 6.

By the use of curves and mechanical devices various methods are described for dividing an angle into any number of equal parts.

2. Dreiling, Lillian. *The Fusion of Plane and Solid Geometry*. High Points (Board of Education, New York City). September 1934. Pp. 64–66.

A clear restatement of the arguments presented both by the opponents and the adherents of the "fusion" movement. Three different methods are proposed for fusing plane and solid geometry and the advantages of each are indicated.

3. Fuhr, H. *Konstruktion mit dem Zeichenwinkel*. (Constructions possible with a triangle.) Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht aller Schulgattungen, 65: 279–87. 1934, no. 6.

The author's thesis is that all geometric constructions yielding equations up to and including the fourth degree can be effected by the use of a drawing triangle alone.

This is an interesting extension of the geometric researches initiated by Mascheroni in the eighteenth century when he proved that all constructions possible by the combined use of compasses and straight edge can be done by compasses alone.

4. Resnick, A. C. *A Study of the Likes and Dislikes of Pupils Studying Plane Geometry*. High Points (Board of Education, New York City). September 1934. Pp. 47–49.

Result of unsigned replies to questionnaires returned by 565 students of Boys' High School, Brooklyn, N. Y. The conclusions are: (1) only 15% of the pupils definitely dislike geometry; (2) the liked topics are, in general, the more interesting in application and in some respects the easier; (3) the book theorems were the third most disliked topic; (4) construction work seems to be very popular; (5) there is a strange unanimity about the likes and dislikes for certain topics of geometry.

**Miscellaneous**

1. Blank, Laura. *Objective Testing in Secondary School Mathematics*. School Science and Mathematics. 34: 702–08. October 1934.

The various types of objective tests available for algebra are described, and their specific functions, advantages and weaknesses are indicated: completion-test, alternative or true-false test, multiple choice, matching test, incorrect statement, analogy test or incomplete proportion, and the continuity or rearrangement test. The applicability of each of the above types to geometry testing is pointed out, and the difficulties inherent in testing specific geometric abilities are considered.

". . . in the matter of testing it is not a question as to whether the essay or the objective test is the better and then making exclusive use of that type. It is rather a question of determining the occasions and circumstances under which each is more valuable and then of using each accordingly. Each of the types has its peculiar merits and advantages; each has its limitations, often quite marked, for a particular subject and a unique aspect of that subject. Employ for that subject, and that particular phase of it, the type of test which fulfills the desired ends."

2. Giles, Catherine. *Not Mathematically Minded*. Journal of Education. 117: 63–4. February 1934.

A humorous account of the travails of a non-mathematically minded person in the realms of checkbooks, budgets, tax blanks, and linoleum floors.

3. Hamley, H. R. *The Function Concept in School Mathematics.* The Mathematical Gazette (London). 18: 169–79. July 1934.

A paper read at the annual meeting of the British Mathematical Association, January 5, 1934. It is in the main a résumé of the author's "Relational and Functional Thinking in Mathematics" which was published as the ninth yearbook by the (American) National Council of Teachers of Mathematics.

4. Hartung, M. L. *Teaching the Scientific Method in Mathematics Classes.* School Science and Mathematics. 34: 596–600. June 1934.

The possibilities of teaching scientific method in mathematics are shown by examples taken from arithmetic and algebra. "The course in algebra should be a vehicle for training in generalization and therefore should be taught inductively practically from start to finish."

The author overlooks the much richer opportunity that geometry affords for training in scientific procedure. Moreover he uses the latter phrase as if it were synonymous with inductive generalization. This conception is much too narrow.

5. Wren, F. L. and Moncreiff, Ruth. *A Suggested Course of Study for Junior High School Mathematics.* School Science and Mathematics. 34: 724–32. October 1934.

The authors accept the following four principles as the underlying philosophy of the junior high school: (a) articulation; (b) exploration, revelation and guidance; (c) interpretation of environment; and (d) motivation. They also relied upon the following sources as a guide to the formulation of the proposed course of study:

(a) an analysis of units found in eight series of modern junior high school texts,
(b) an analysis of units found in fourteen recently revised courses of study gathered from all sections of the United States,
(c) the Report of the National Committee on Mathematical Requirements, and
(d) Schorling's "A Tentative List of Objectives in the Teaching of Mathematics."

The course of study is presented in great detail in the form of units of subject matter. A columnar arrangement enables the reader to determine to what degree the previously mentioned authorities have concurred upon the inclusion of any given topic in the proposed syllabus.

The authors claim the following features of their course of study as worthy of special note:

(a) "Adequate provision has been made for maintenance of and remedial work in the fundamental skills of arithmetic.
(b) "The value of checking is emphasized throughout the course of study.
(c) "Provisions made for lessons in appreciation and history make possible a further development of the scope and power of mathematics. Such provisions along with optional material and supplementary exercises afford ample opportunities for taking care of individual differences.
(d) "Furthermore the effort is made to relieve mathematics of the stigma of a mere 'tool subject' and to elevate it above the level of manipulative mechanics and to place it in its proper setting as a fundamental mode of thinking."

# Some Mathematical Shortcomings of College Freshmen*

*By* Alan D. Campbell

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When I was teaching mathematics at Northwestern University I had to correct and hand back the homework of my students. I soon wearied of making the same correction on the paper of each student and sometimes over and over again on the papers of the same student. Besides, the students often just threw away their returned homework without looking it over.

I decided to imitate the English instructors and make out a list of errors to be mimeographed. Then I could put a number on a student's paper beside a problem that was wrong and let the student look up the error in his copy of the mimeographed list. Whereas the English hand-book of errors had several hundred listed, I found about 120 errors from freshman mathematics clear through the calculus.

Unfortunately, I lost the list. I remember, however, that of this list only a few appeared again and again. These few errors were mostly in arithmetic, algebra, and trigonometry. Many students fail the calculus not because they do not understand it, but because they cannot add, subtract, multiply, and divide correctly.

Some errors show that the student has only a formal knowledge of mathematics. If he attempts to solve $3x=4$ and obtains $x=-4/3$ or even $x=4-3$ it would seem that he is trying to remember a rule he has once memorized but not understood and that he is not using common sense.

I tell my students that mathematics is only organized common sense. Perhaps that is why mathematics is so hard to learn, because common sense is so uncommon among human beings that it is often called horse sense.

The above error due to a formal knowledge of the subject reminds me of the story of a man who was doctoring himself from a medical book he got from the library. Finally, he became so ill he just had to go see a doctor. When the doctor found out what

* An address given before the Mathematics Section of the New York State Teachers Association, Central Zone, at Utica on Oct. 26, 1934.

the man had been doing he said, "Look out, or some day you will die of a misprint."

I should like to have a dollar for every time I have seen $-(a+b)$ written as $-a+b$. I try to make the students think of $-(a+b)$ as $-1(a+b)$ so as to correct this error. Here I should like to ask how you can break a student of making this sort of mistake. It is like breaking a bad habit. Sometimes I threaten to arrange to give a student an electric shock every time I have to upbraid him for that sort of error.

I often put on the black-board a list of errors the students must avoid or, after warning them what I am going to do, I work a problem putting in all the errors I have seen them make. However, this sometimes fails because they imitate the errors. In the army they say an order should read positively such as "Continue beyond the spot A" and never negatively such as "Do not stop at the spot A." This is because the word "not" is so often overlooked or forgotten and the order remembered erroneously as "Do stop at the spot A." I wonder if this advice does not apply in the teaching of mathematics.

The sergeants in the regular army used to tell me they would rather drill a raw recruit than one who had been in a poor national guard outfit, because the latter recruit would have learned bad habits of drill and discipline that must first be cured before correct habits could be formed. I sympathize with these sergeants when I run across a student who cannot do even simple arithmetic and apparently had a bad start away back in the grades. On the other hand it is a pleasure to find a student who has been well taught by someone who also instilled in him a love for mathematics.

A teacher told me about wrestling with a student over the solution of a cubic equation. The student wanted a formula that would tell him what values of the variable to test out in the equation in the hunt for roots. When he heard there is no such formula, the student said in a disgusted tone, "Then this isn't mathematics, because you haven't any formula and all you can do is to think it over and use common sense." This story illustrates beautifully the average student's attitude toward mathematics and also the fact that it is so hard for him to think a problem through and use common sense that he wants to have some formula to use blindly. Such a student is like a crazy carpenter who wants to saw a board

and shuts his eyes, puts his hand in his tool-chest, and uses the first tool his hand touches.

One professor told me that freshmen are not superstitious because they do not believe in signs. What a pun! I claim that many freshmen are superstitious because they come to college with the idea that mathematics is too hard for them, that they never can learn the subject. Who gave them this idea, their teachers or their parents? The only students who cannot learn mathematics are those who have no intelligence or common sense at all. One college girl had to make up an entrance condition in plane geometry. She put this off year after year because she dreaded it. Finally she had to be tutored in plane geometry and she found she enjoyed it and really had quite an aptitude for it.

I asked a professor of chemistry where his students were weak in mathematics. He said "in pointing off decimals, in handling ratios, in percentages, and in addition." I asked the same question of a professor of physics. He told me that students have never learned to express the conditions of a problem by suitable equations. Then he gave me a copy of The American Physics Teacher Vol. 2, No. 1, Feb. 1934. Herein was an interesting article by a professor of education at the University of Iowa entitled "Student Disabilities in the Mathematics of First-Year College Physics." He finds that the students have trouble with positive, negative, and fractional exponents, with linear equations involving decimal coefficients, with factoring in algebra, and with handling fractions.

On the basis of a careful study made at Iowa, a work-book entitled "Review of Pre-College Mathematics" by Lapp-Knight-Rietz has just been published by Scott, Foresman and Company. I wrote Professor Rietz of Iowa and he sent me a copy of the Iowa Placement Examination in mathematics with comments by himself and Professor Woods.

The comments of Professors Rietz and Woods seem to indicate that much of the students' trouble in mathematics comes from not understanding what they read. Their vocabulary is meagre. They do not know what such words as per cent, product, quotient, sum, and consecutive really mean. These students tell me that in high school they never read the textbook, that the teacher always explained the next lesson, worked some typical examples, and then gave them the same sort of examples for home-work.

At the first meeting of my freshman class I usually read the first

assignment to the class, explaining the meaning of the long words. At the next meeting I read the first assignment with the class. At each meeting for a week or so I read the text with the class, after they are supposed to have read it by themselves in preparation for the day's lesson. By this means I hope to get them to read and understand the text. They must learn not to read "discriminant" for "determinant," "graft" for "graph," "factors" for "roots." They must learn how to obtain information from books and periodicals. It is amusing but pitiful to have students think that i.e., q. e. d., e. g., and viz. are parts of formulas or else not to have any idea what these abbreviations mean and to decide to ignore them. I asked an engineer what *BVD* meant. He said he thought *V* meant velocity, *D* was distance, but that *B* had him puzzled.

This reminds me of a story about Arkansas. The state legislature had passed a bill prohibiting the teaching of evolution in the schools. The janitor of one high school reported to the principal that the professor of mathematics was breaking the law. It turned out that the janitor had overheard the teacher of mathematics talking about involution and evolution in algebra. I wonder if from then on evolution was termed root extraction. My students often smile at the word depression in the term angle of depression and I threaten to change the name to angle of down-sight.

Many students will not ask questions in class. They think they are so much stupider than the rest of the class, or they do not want to slow up the class, or they plan to ask me about it afterwards during my office hours, or they decide they will omit any question on that point that may appear in the examination. When I have explained something of importance, I ask each and every student in turn if he understands it. Sometimes, even when he says he does understand it, he is prevaricating or mistaken, so I give him a problem to test his knowledge of it. In a student's mind there is a great difference between understanding a proof the teacher gives and being able to reproduce the proof and to use the formula or theorem proved in the solution of a problem.

In the grades or in high school these students must have been prevented from asking questions in class. The class in mathematics seems to me to be like a laboratory. How can the student handle a mathematical tool or instrument without asking questions about it in class?

In high school the subjects of logarithms and of determinants

are often just touched upon and the students are told to omit questions on them in the Regents' Examination. Unfortunately, these are most important topics for college use. Also such a simple geometrical fact as that any two angles whose sides are mutually perpendicular are either equal or supplementary is not stressed in plane geometry. This fact is of the utmost importance in trigonometry and analytics. *Would it not be well for teachers of mathematics in the grades as well as those in high school to have had mathematics through the calculus so as to see the importance for later mathematics of the different topics they teach?*

It is amusing to see how a college student on an examination follows the rules of mathematics in working a problem until he is stuck, then he pitches these rules overboard, and sometimes obtains the correct answer by all kinds of erroneous methods. The students' lack of clarity in thinking is reflected in his statement of problems. For example, one student wrote let $x$ = John (instead of the number of years in John's age five years ago) and let $y$ = Henry. Now I ask you, what does $x-y$ mean?

Here is a brief list of common errors: $3/4\times 2/3=9/8$; $2^3\times 2^4=4^7$; $7/8+3/4=4.7+3.8$; $\sqrt{9+16}=3+4$ and $\sqrt{a^2+b^2}=a+b$; $9/10=\dfrac{7+2}{7+3}=2/3$; $3x-2y=7$ is the same as $6x-4y=7$; $(x-3)(x-2)=4$ gives $x-3=4$ and $x-2=1$; $x^2=4$ gives $x=+2$; $5\sqrt{3}+6\sqrt{2}=11\sqrt{5}$; $\sqrt{-4}=-2$; $\dfrac{\log a}{\log b}=\log a-\log b$; $x^2+y^2+2x-4y=1$ is the same as $x^2+2x+1+y^2-4y+4=1$.

Solving a quadratic equation is most important in college mathematics and its applications, so also is completing squares, and putting such an equation as the last one above into the form $(x-h)^2+(y-k)^2=r^2$ and then interpreting $h, k$, and $r$ geometrically.

Students cannot even substitute into formulas correctly. The mere algebraic handling of equations, clearing of fractions and so forth, trips up many students in the study of trigonometric equations. Furthermore such an equation as $2\cos^2\theta+3\cos\theta+1=0$ does not look to them like $2x^2+3x+1=0$. Many high school teachers allow students of trigonometry to work with both sides of an identity. This destroys all the value of the identity as a training in reducing complicated expressions to simpler forms.

Students cannot derive formulas. They do not use the lowest common denominator when combining fractions. They have no idea of what a function means and when they see cos $(a+b)$ they at once write cos $a+$cos $b$ as though they were clearing out the parentheses. Students do not even hear clearly. One student heard a teacher speak of De Moivre's Theorem until finally he asked, "When are you going to teach us tomorrow's theorem?"

When I was in college my algebraic and arithmetical operations were flawless, but now I seem to commit all the errors I have noticed and corrected so often on students' papers. Teachers of English tell me they notice the same sort of deterioration in their spelling, grammar and punctuation.

I want to quote here what a student told me was called in his high school the Golden Rule for equations namely "Do unto one side what you have done unto the other." I want to tell you about a student who could not understand why $a^0=1$ and not $a^0=a$, why $a\times 0=0$ and not $a\times 0=a$, and why $a/0$ is not $a$. He argued that $a^0$ is raising $a$ to the zero power, $a\times 0$ is multiplying $a$ by zero, and $a/0$ is dividing $a$ by zero, so that $a^0$ is not raising $a$ to any power, $a\times 0$ is not multiplying $a$ by anything, and $a/0$ is not dividing $a$ by anything. Therefore $a^0=a$, $a\times 0=a$, and $a/0=a$. I told this student to take up law and not mathematics.

My freshmen are very much amused over what I call my search for a key man in each class. By a key man I mean a student who is so poor in mathematics that if I can make him understand my explanations I can feel sure that everyone in the class understands.

I believe the sources of the mathematical shortcomings of college freshmen can be found in the grades. I feel these shortcomings should be tabulated carefully and analyzed to see what causes them, and then a way sought to cure them. In Europe and in some places in the United States the calculus is taught in the schools, but for the most part in the United States we have to go back and review arithmetic and algebra in college. More and more mathematics must be learned in the future by college students who plan to go into mathematics or physics or to take up some other field of application of mathematics. This will mean the crowding of more mathematics down into the freshman year and even into high school and into the grades. Therefore we must somehow do away with this having to repeat so much elementary material in college and this slowing up due to poor handling of arithmetic and algebra.

# NEWS NOTES

MATHEMATICS section meetings as a part of the programs for the Nebraska State Teachers Association were held Oct. 25 and 26 in each of the six districts as follows.

*District Number One at Lincoln*

Demonstrations of class room teaching of mathematics classes in both junior and senior high school at the Everett Junior High School directed by C. L. Culler, principal of the Whittier Junior High School of Lincoln, Thursday Oct. 25.

*Mathematics Section—Friday 3 P.M.*

*President:* Gladys E. Van Camp, Lincoln

*Secretary:* Esther Hall, Fairbury

*Address:* A Fundamental Strategy—Dr. F. B. Knight, University of Iowa

*District Number Two at Omaha—Friday 2 P.M.*

*President:* H. A. Campbell, Technical High, Omaha

*Secretary:* Clara Weyrick, Plattsmouth

Program dealing with advantages and problems to be considered in connection with having mathematics classes composed of students selected by prognostic tests of their mathematical aptitude.

"The Present status of the Prognostic Test," Dr. A. R. Congdon, University of Nebraska.

"Problems of Administration from the Point of View of the University Official," Dean William Thompson, Municipal University of Omaha.

"The Probable Effect Upon the Work of University Classes," Dr. J. M. Earl, Municipal University of Omaha.

"Problems of Administration to be Solved by the Officials of the Smaller High Schools," Supt. Galen Saylor, Waterloo.

"The Advantages as Seen from the Standpoint of the High School Teacher," Miss Grace Fawthrop, Central High, Omaha.

*District Number Three at Norfolk—Friday 10:30 A.M.*

*Chairman:* Miss Enid Conklin, Wayne State Teachers College

"Putting Our House in Order in Mathematics," Dr. Harl R. Douglass, University of Minnesota.

Group discussion.

*District Number Four at North Platte—Friday 1:30 P.M.*

*President:* Clarence Lindahl, Paxton

*Secretary:* Mary Hoagland, Hastings

"Value of Mathematics in Our Present Day Curriculum," Dr. W. C. Brenke, University of Nebraska.

*District Number Five at Mc Cook—Thursday 3 P.M.*

SCIENCE AND MATHEMATICS

*Chairmen:* Herbert W. Finke, Holdrege, Science; D. C. Scofield, Trenton, Mathematics.

"Recent Practical Accomplishments in the Field of Science and Mathematics," Prof. C. J. Frankforter, University of Nebraska.

*District Number Six at Chadron—Thursday 1:30 P.M.*

*President:* Frank Barta, Lisco

*Secretary:* Kenneth A. Rawson, Dix

PROGRAM

"Classroom Problems," Kenneth Rawson, Prin. Kimball County High School

Address: "The Inside of Story 2 Plus 2," Dr. F. B. Knight, University of Iowa

Business Meeting

"High School Objectives of Mathematics," Supt. W. L. Nicholas, Oshkosh

The above programs were assembled by Prof. A. L. Hill, Peru State Teachers College, Peru, Neb.

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Dr. Hans Rademacher, formerly professor of mathematics at the University of Breslau, Germany, has joined the faculty of the University of Pennsylvania for one year under a joint grant from the Emergency Committee in Aid of Displaced German Scholars and from the Rockefeller Foundation.

Dr. Rademacher, who is one of the leading younger mathematicians of Germany and who served on the faculty at the University of Hamburg before going to Breslau, is noted particularly for his research work in the analytic theory of numbers, differential geometry, and the theory of functions of a real and complex variable.

According to Dr. H. Lamar Crosby, dean of the Graduate School at Pennsylvania, Dr. Rademacher will conduct a seminar course in the Graduate School in one of the fields of his research.

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The Mathematics section of the Fourth Annual Teacher Training Conference gave the following program at North Texas State Teachers College at Denton, Texas:

Morning Session

Miss Mary Ruth Cook, Presiding
(Discussion following each address)

"Formulating the Objectives of Mathematics Teaching for the New Age"—Mr. E. D. Box, East Texas State Teachers College

"How are the New Objectives of Mathematics Teaching to be Achieved" —Miss Bessie Penick, Graford, Texas, Mr. James F. Sartain, Garland, Texas.

Afternoon Session

Mr. A. S. Keith, Presiding
(Discussion following each address)

"What Are the Characteristics of the Progressive Mathematics Teacher"—Miss Elizabeth Dice, Dallas, Texas.

"How Can This Institution Effectively Educate the Mathematics Teacher of Tomorrow"—Mr. Amos Barksdale, Denton, Texas.

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The Ninth Annual Conference of Teachers of Mathematics was held at Iowa City, Ia. On October 19th and 20th. The following program was given:

FRIDAY MORNING,
OCTOBER 19, 1934

*North Room, Old Capitol*
H. L. RIETZ, *presiding*

Address: Procedures Effective in Teaching Dull Pupils. RALEIGH SCHORLING, University of Michigan.

Address: Astronomy in the Junior High School. C. C. WYLIE, University of Iowa.

Address: An Attempt at Solving the Problem of Individual Differences. CLARA D. MURPHY, Evanston Township High School.

Discussion

FRIDAY AFTERNOON,
OCTOBER 19, 1934

*North Room, Old Capitol*
L. E. WARD, *presiding*

Address: The Contribution of Arithmetic to the Progress of Mankind. L. H. WHITCRAFT, Ball State Teachers College

Address: Some Phases of the Problem of Teaching Mathematics to Dull Pupils. RALEIGH SCHORLING.

Discussion

FRIDAY EVENING,
OCTOBER 19, 1934

*Iowa Memorial Union*

Conference Dinner

ROSCOE WOODS, *toastmaster*

"Panel" discussion on "The Value of Mathematics in the High School Curriculum" by Ruth Lane, assisted by the following "Jury:"

Raleigh Schorling, Allen T. Craig, J. W. Querry, Gertrude Herr, H. V. Price, Dora E. Kearney, Mae Howell, Clara D. Murphy, L. H. Whitcraft, W. E. Beck, Ruth Balluff, H. L. Rietz.

SATURDAY MORNING,
OCTOBER 20, 1934

*North Room, Old Capitol*

J. F. REILLY, *presiding*

Address: Some Suggestions for Meeting Pupil Differences. CLARA D. MURPHY.

Address: Taxonomy of Mathematics. E. W. CHITTENDEN, University of Iowa.

Address: Pictures as an Aid to Learning Mathematics. L. H. WHITCRAFT.

# NEW BOOKS

*Fusion Mathematics*. By Aaron Freilich, Henry H. Shanholt, and Joseph P. McCormack. Silver, Burdett and Company, 1934, VII+600. Price, $1.84.

This book is intended to cover the essential features of the courses in intermediate algebra and trigonometry and is planned for use in the eleventh grade of the senior high school. Moreover, it is written on the fusion plan so as to avoid the repetition of such topics as exponents, logarithms, and radicals which in our traditional courses are not only repeated, but are not presented in such a way that the pupil can make immediate use of the topic in one field as soon as it is presented in the other.

The fusion idea is essentially sound and there is no place in the secondary school where fusion is more easily carried out than in meaning algebra and trigonometry together. The order of treatment of the topics in this new book is not the only method of organization nor do the authors claim that it is necessarily the best, but it provides an interesting way of trying out the fusion idea and it should be of interest to teachers generally. The authors of the book are all experienced classroom teachers of mathematics who have brought together in this one Volume the best of their combined thoughts on the teaching of the two subjects presented.

The book is written primarily to the student. Each new idea is motivated by showing the pupil the need of a particular topic. Dependence, Variation and relationships are used as unifying principles.

The book is long, but this need not be a drawback if teachers realize the need for meeting the individual differences among the pupils with respect to their varying needs, interests, and abilities. A book should contain enough material of sufficient difficulty to test the best pupils in the class and also material that is easy enough to be done by the slower pupils. From such work it is the duty of the teacher to select what is best fitted to his local situation.

This is not the first book of its kind, but it is another bit of evidence to show that the teaching of mathematics in separate compartments is on the way out in so far as the junior and senior high schools are concerned.

*Intermediate Algebra*. By Aaron Freilich, Henry H. Shanholt, and Joseph P. McCormack. Silver, Burdett and Company, 1934, IX+406. Price, $1.40.

This book contains essentially the same material in intermediate algebra that is included in the fusion book just reviewed above. It is doubtless intended by the authors for use in those schools where the teachers are not yet ready for the fusion idea. Aside from the value of having algebra fused with trigonometry the treatment is much the same. However, teachers should realize the value of the fusion course where when an idea or a topic is once presented carefully it does not have to be repeated as is done in our traditional treatments of algebra and trigonometry.

*Plane Trigonometry*. By Aaron Freilich, Henry H. Shanholt, and Joseph P.

McCormick. Silver, Burdett and Company, 1934, IX+293. Price, $1.32.

Here again this book is essentially the same course in plane trigonometry that is included in the fusion course received above. Some teachers, even those favorable to fusion, may prefer to use this book together with the intermediate algebra book reviewed above rather than to use the fusion book so that they can do their own fusing so to speak. This, however, would seem to be the exception rather than the rule. There will still be schools which will prefer the separate volumes and for them these books will be of interest.

# THE MATHEMATICS TEACHER

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This organization has for its object the advancement of mathematics teaching in the elementary school and in junior and senior high schools. All persons interested in mathematics and mathematics teaching are eligible to membership. All members receive the official journal of the National Council—THE MATHEMATICS TEACHER—which appears monthly except June, July, August and September.

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